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Analytical Solution of Two-dimensional Heat Equation in the Context of COVID 19 using Dirichlet Boundary Condition

M. B. GREMA[1], Modu Mustapha Tijjani[2], H. I. SALEH[3] and Sadiq Alhaji Goni[4]

[1]Department of Mathematics and Statistics, Ramat Polytechnic Maiduguri, Borno state. Nigeria P.M.B 1070

[2]Department of Computer Engineering, Ramat Polytechnic Maiduguri, Borno state. Nigeria P.M.B 1070

[3]Department of Computer Science, Ramat Polytechnic Maiduguri, Borno state. Nigeria P.M.B 1070

[4]Department of Electrical and Electronics Engineering, Ramat Polytechnic Maiduguri, Borno state. Nigeria P.M.B 1070

Abstract: *The study focused on analytical solution of two- dimensional heat equation with Dirichlet boundary condition in the context of COVID 19 , by method of separation of variable of a partial differential equation which is transformed to second-order differential equation and double Fourier series was used to obtain the analytical result. Also, the result of the analytical solution was used applied in the context of COVID 19, the result obtained in problem1 shows that the graph is moving downward on the value -1.98 and 0.25 and maintained a straight line, while in problems 2 the result shows that the graph is moving upward on the values 0.99 and 0.001 and also maintained a straight line. This shows that COVID 19 has the highest level in problem 2 than 1 because the heat has a significant effect on problem 2. Analytical solution of two- dimensional heat equation is time-consuming and also is simpler, efficient, and precise; hence the study recommended that the method should be applied into a multidimensional heat transfer equation in the context of COVID 19 with a view to solving scientific and engineering problems.*

Key words: *2 dimension, heat equation, COVID 19 and Dirichlet Boundary Condition*

1. Introduction

Two dimensional heat equations.

$$u_t = k\left(u_{xx} + u_{yy}\right)$$

Related formulae hold:

$u = \iint G_2\left(x, y; x', y', t\right)\left(x', y'\right)dx'dy'$ with $G_2\left(x, y; x', y', t\right) = G_1\left(x, x'\right) = G_1\left(y, y', t\right)$ (ivrii, 2017)

Study of some Selected uniqueness results for the singular heat equation is obtainable, the study also reveals by Eigenfunction of some problems that remain effective (young, 1984).

Pot-Pot refrigerator was solved by explicit and implicit numerical procedures, the end result was compared and it reveals that the numerical method used is more natural than the analytical techniques. it has been observed that the outcome fit very well with predictable evolution system (Nalasco, Jacome and Hurtado-Lugo , 2017).

An optimal control problem for the heat equation with phase change taking latent heat of fusion in to account has been obtained. The outcomes for the input function is a series of step function and a smooth function can be found to approximate this optimal solution, but its result most likely lead to suboptimal solutions (Josef ,2016).

Kurt (2008) investigate the study for a square interms of elliptic functions, the numerical result has been computed and compared by the technique of separation of variable and finite element. The study also reveals that the technique gives exact result than the others method in the solution of the two-dimensional heat equation.

Uba, Grema and Mai (2019) examine the solution of one-dimensional heat equation using residue calculus; the study investigates that method of separation of the variable. It has been found that residue theory was also used to solve the second other differential equation after separation of variables, residue calculus is time-consuming and it is simpler, effective and accurate. The study also reveals that the method can be extended into multidimensional heat equations. The method also can be compared with the other numerical method.

The heat equation is pretentious second-order partial differential equations (PDE) that describes the variation in temperature in a specified area over a while. Heat, is a vital problem in many physical disciplines including science and technology, is an energy transit and it is a branch of thermodynamics which deals with the amount of heat transfer among two or more equilibrium state of a system in a medium or between media. (Dabrat, kapoor and Dhawan , 2011 and Suresh 2018).

Finite element-finite difference method has been established for solving parabolic two-step micro heat transport equations in a three-dimensional double-layered thin film exposed to ultrashort-pulsed lasers, It is revealed that scheme is unconditionally stable for the heat source and the numerical results for thermal analysis of a gold layer on a chromium padding layer are found and it's also revealed that the technique can be applied to multiple layers and irregularly shaped geometries (Brian, 2014).

Kutanaei, Ghasemi and Bayat , (2011). The study that the solution obtained from the numerical simulations is compared with those obtained by the finite volume (FV) method. The results show that the present method is in very good agreement by finite volume method and this is due to the fact that the radial basis function-based differential quadrature (RBF-DQ) process is an exact and flexible method in the solution of heat conduction problems.
A comparative study between the Fourier transforms and Wavelet transforms for the one-dimensional heat equation was studied, (Husein and Alaa, 2016).

A solution of one-dimensional Heat Equation by the method of separation of variables using FOSS tools Maxima was used. The results obtained by the separation of variables are the same as the results achieved by using Maxima program (Sudha, Geetha and Harshini, 2017).
Samaneh and Soltanalizadeh (2014) assert that the solution of the second-order two space dimensional diffusion equation by using the differential transformation method. Changing the model of the partial differential equation into a system of linear equations and computational difficulties of the other methods can be abridged by applying this method and also made emphases only in the solution of two-dimensional inhomogeneous diffusion equations subject to a nonlocal boundary condition.
Douglas equation was used to achieve implicit finite-difference equations for two-dimensional heat- transfer equations, also accuracy was observed by the Fourier series process for stability analysis (Gülkaç , 2015).
Adi and Alexander (2008) investigate that the Boundary condition for the case in which the heat equation is satisfied outside the domain of interest with no limitations on the equation inside was developed

Johansson, B.T. (2016) examine that the result of Properties of a method of fundamental solutions for the parabolic heat equation leads naturally to a process of numerically approximating solutions to the parabolic heat equation represented a system of fundamental solutions (MFS)and a discussion around the convergence of such an approximation was built-in.

comparative study between the analytic solution and numerical solution of one-dimensional heat diffusion equation focus to Robin boundary conditions multiplied by a small parameter epsilon greater than zero, the numerical result reveals that the numerical solution of the differential equation with Robin boundary condition is very close by in certain sense of the analytic solution of the problem through homogeneous Dirichlet boundary conditions when ε tends to zero (LOZADA-CRUZ, RUBIO-MERCEDES AND RODRIGUES-RIBEIRO, 2018).
Elliptical domain has been made with the governing of two dimensional (2D) heat equation that is discretized with the Finite Difference Method (FDM) and the stability condition has been defined and the numerical result by writing MATLAB codes has been found with the stable values of time-domain (Mehwish, Asif and Shakeel, 2020).
Saeed (2015), Investigate the analytical and numerical solution of one-Dimensional a rectangular Fin with an Additional Heat source. The study also investigated the influence of the heat source on the temperature profiles and the fin efficiency was discussed in the case of the heat source $\psi = 0$ for the wide range of parameters (M).

Analytical and numerical methods were used in the correlation of the solution of two-dimensional steady-state heat conduction, and the result obtained shows that the finite element is in good agreement with the analytical values and solution reveals that it can be effectively used to more complex thermal problems (Suresh 2018).

The theory of heat equations was first established by Joseph Fourier in 1822; Heat is the dynamic energy of particles that are being exchanged and is associated with the study of Brownian motion. The one, two, and three-dimensional wave equation was revealed by Alembert and Euler. The solutions of heat and wave equations have attracted the attention of various authors in mathematics, such as the optimal homotopy asymptotic method (OHAM), the modified Adomian decomposition method (MADM), the variational iteration process, the differential transform method (DTM), the Homotopy perturbation method (HPM) (Hassan, Rasool, Poom and Arif 2019).

Most of the study mentioned above did not apply the Analytical method in the Solution of Two-dimensional Heat Equation, in order to solve problems in the Context of COVID 19 using Dirichlet Boundary Condition. But they applied many methods and numerical techniques to solve it. The objective of this paper, is therefore to determine the application of the analytical method in the solution of two-dimensional heat equation in order to solve problems in the contex of COVID 19 using Dirichlet-Boundary Condition.

2. Methodology

$$\frac{\partial T}{\partial t} = \kappa\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial y^2}\right)$$ We put $T(x,y,t) = X(x)Y(y)\tau(t)$

$$\frac{1}{X}\frac{d^2X}{dx} + \frac{1}{Y}\frac{d^2Y}{dy^2} = \frac{1}{\kappa\tau}\frac{d\tau}{dt} = -\mu^2(say)$$ Where μ^2 is separation constant, μ be real

Then $\tau = A_1 e^{-\kappa\mu^2 t}$

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\left(\frac{1}{Y}\frac{d^2Y}{dy^2} + \mu^2\right) = -p^2(say)$$

$$\frac{d^2X}{dx^2} + p^2X = 0 \quad and \quad \frac{d^2Y}{dy^2} + q^2Y = 0$$

$X = A\cos px + B\sin px \;\; and \;\; Y = C_1\cos qy + D_1\sin qy$ and $q^2 = \mu^2 - p^2$

$$T(x,y,t)=(A\cos px+B\sin px)(C_1\cos qy+D_1\sin qy)A_1e^{-\kappa\mu^2 t}$$

$$T(x,y,t)=(A\cos px+B\sin px)(C\cos qy+D\sin qy)e^{-\kappa\mu^2 t}$$ Where $C=C_1A_1,\ D=D_1A_1$

The boundaries of the rectangle $0\le x\le a,\ 0\le y\le b$ are preserved at zero temperature. If at t = 0, the temperature T has prescribed value f(x,y) show that

t >0, the temperature at a point within the rectangle is given by

$$T(x,y,z)=\frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}f(m,n)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{b}\right)e^{-\kappa\mu_{mn}^2 t}$$ Where

$$f(m,n)=\int_0^a\int_0^b f(x,y)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{b}\right)dxdy$$ and $\mu_{mn}=\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)$

Solution

$$\frac{\partial T}{\partial t}=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right),\quad 0<x<a, 0<y<b, T>0$$

$$T(x,y,0)=f(x,y),\quad 0<x<a,\ 0<y<b,\ t>0$$

$$T(0,y,t)=f(a,y,t)=0\qquad 0<y<b,\quad t>0$$

$$T(x,0,t)=f(x,b,t)=0\qquad 0<x<a,\quad t>0$$

The solution of the equation is

$$T(x,y,t)=(A\cos px+B\sin px)(C\cos qy+D\sin qy)e^{-\kappa\mu^2 t}$$ Where $\mu^2=p^2+q^2$

$$T(x,y,t)=0\ \ and\ T(x,0,t)=0$$

$A=0\ \ and\ l=0$ thus $T(x,y,t)=BD\sin px\sin qye^{-\kappa\mu^2 t}$

Also the boundary condition $T(a,y,t)=0$ and $T(x,b,t)=0$, $\sin pa=0$, $\sin qb=0$, $p=\frac{m\pi}{a}$ and $q=\frac{n\pi}{a}$ respectively, where $m=1,2,3,\cdots\ \ and\ n=1,2,3,\cdots$

Hence using the superposition principle

$$T(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)e^{-\kappa\mu_{mn}^2 t}$$ Where $\mu_{mn}^2=p^2+q^2=\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)$

Finally, the given initial condition implies

$$f(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)$$

From this it clearly that it represents a double Fourier series particularly the sine series, to obtain we use orthogonal double Fourier series and so

$$A_{mn}=\frac{2}{a}\frac{2}{b}\int_0^a\int_0^b f(x,y)\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)dxdy$$

Hence the require solution is

$$T(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}F(m,n)\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)e^{-\kappa\mu_{mn}^2 t}$$

$$T(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}F(m,n)\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)e^{-\kappa\mu_{mn}^2 t}$$ Where

$$F(m,n)=A_{mn}=\frac{4}{ab}\int_0^a\int_0^b f(x,y)\sin\left(\frac{mx\pi}{a}\right)\sin\left(\frac{nx\pi}{b}\right)dxdy$$

3. RESULT

Problem 1 : solve the equation $u_t=(u_{xx}+u_{yy})\quad 0<x<\pi,\quad 0<y<\pi\quad t>0$

$$u(x,y,0)=x(\pi-x)y(\pi-y)\qquad 0<x<\pi,\ 0<y<\pi$$

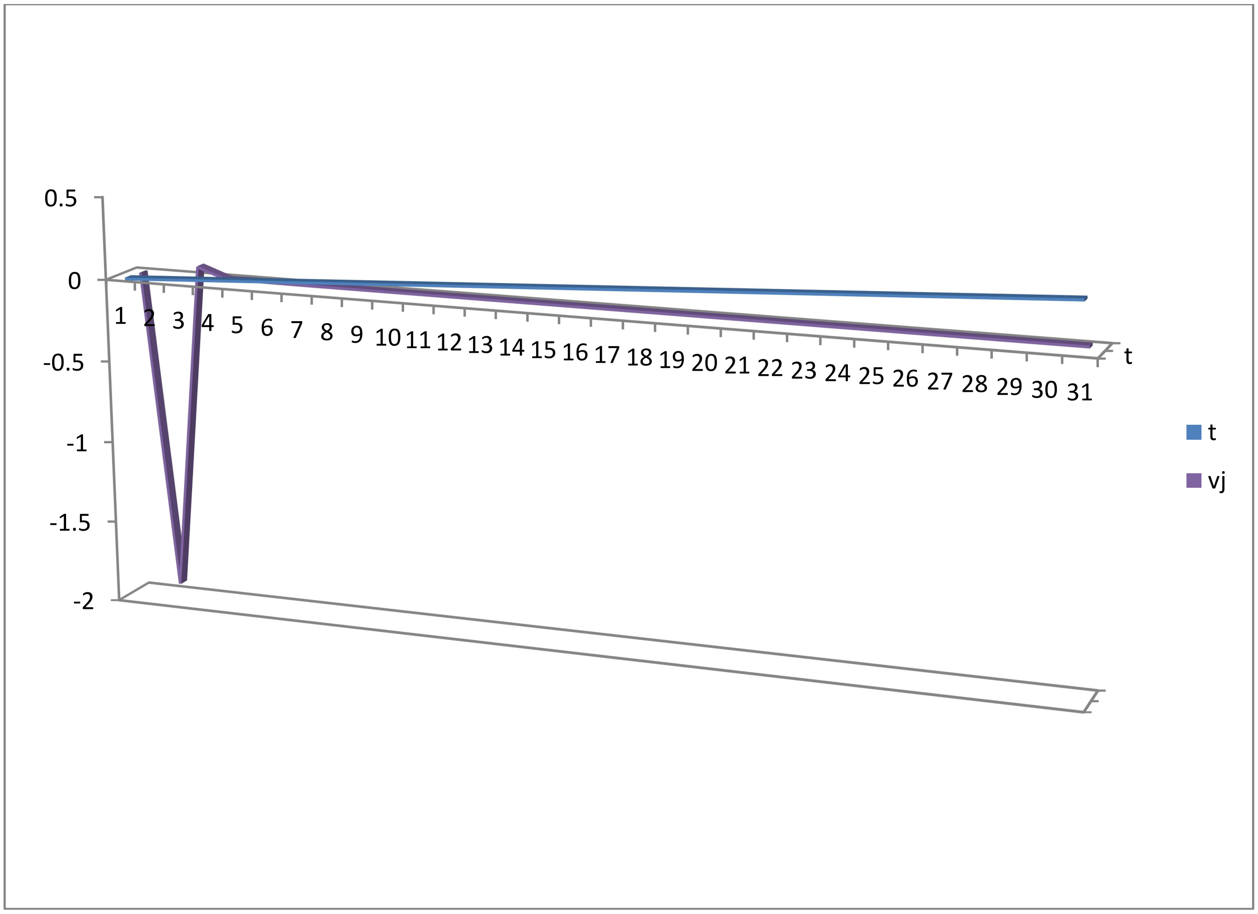
$$u(0,y,t)=u(\pi,y,t)=0,\qquad u(x,0,t)=u(x,\pi,t)=0$$

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}c_{nm}\cos nx\cos mye^{-t\left(n^2+m^2\right)}$$

$$u(x,y,0)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}c_{nm}\cos nx\cos my$$

$$c_{nm}=\frac{4}{\pi^2}\int_0^{\pi}\int_0^{\pi}x(\pi-x)y(\pi-y)\cos nx\cos mydxdy=\frac{4(-1)^{n+m+2}}{n^2m^2}$$ Using double Fourier series

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4(-1)^{n+m+2}}{n^2m^2}\cos nx\cos mye^{-t\left(n^2+m^2\right)}$$



Problem 2 : solve the equation $u_t = (u_{xx} + u_{yy}) \quad 0 < x < \pi, \quad 0 < y < \pi \quad t > 0$

$$u(x, y, 0) = x \quad 0 < x < \pi, \ 0 < y < \pi$$

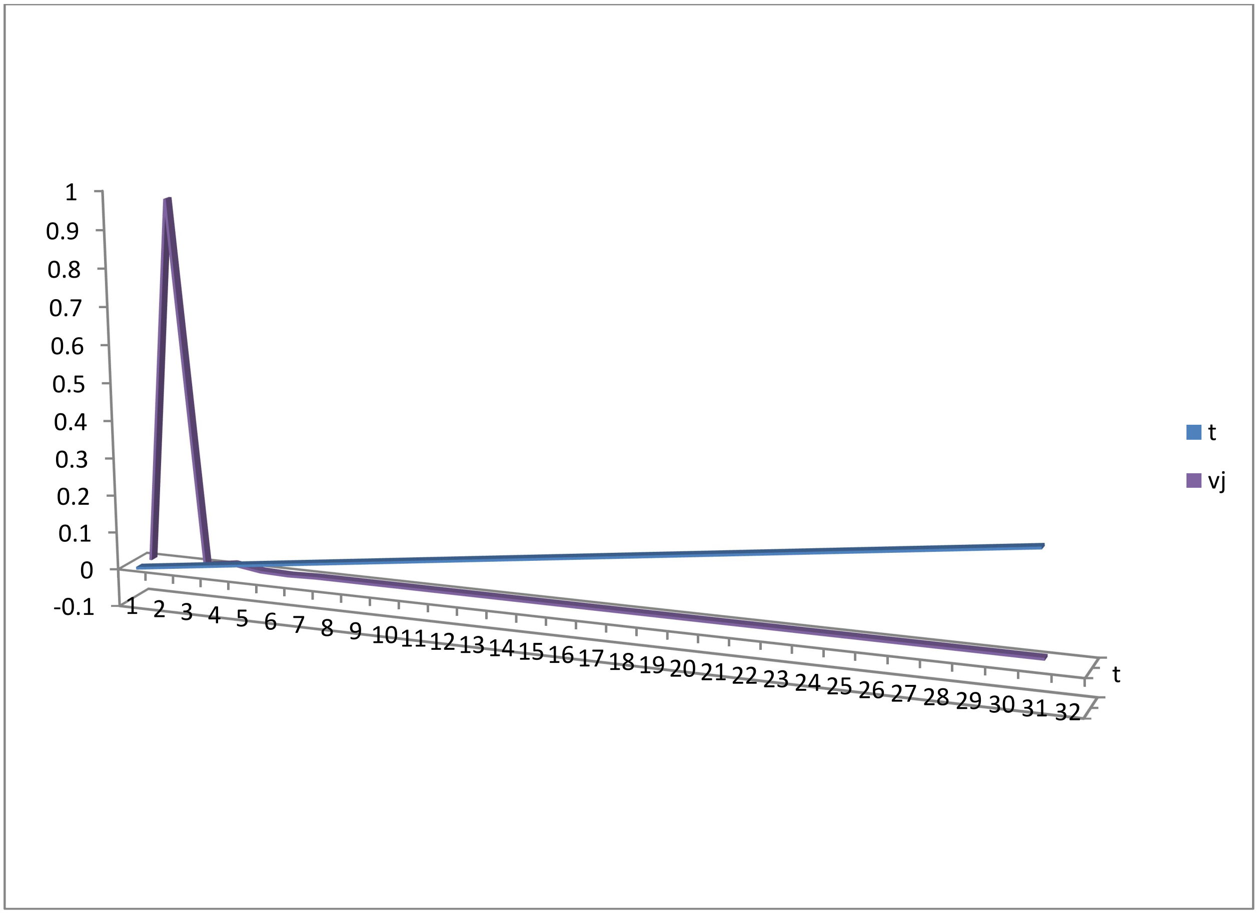
$$u(0, y, t) = u(\pi, y, t) = 0, \quad u(x, 0, t) = u(x, \pi, t) = 0$$

$$u(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} \cos nx \cos my e^{-t(n^2+m^2)}$$

$$u(x, y, 0) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} \cos nx \cos my$$

$c_{nm} = \frac{4}{\pi^2} \int_0^{\pi} \int_0^{\pi} y \cos my dx dy = \frac{4}{m^2} \left[(-1)^m - 1\right]$ using double Fourier series

$$u(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4\left((-1)^m - 1\right)}{m^2} \cos nx \cos my e^{-t(n^2+m^2)}$$



Conclusion

Analytical solution of two- dimensional heat equation with Dirichlet boundary condition in the context of COVID 19 was solved, by the method of separation of the variable of a partial differential equation and transformed to the second-order differential equation and double Fourier series was used to obtain the analytical result. Also the result of the Analytical solution was used to apply in the context of COVID 19, the result obtained in problem 1 shows that the graph is moving down ward on the value -1.98 and 0.25 and maintained a straight line, while in problems 2 the result shows that the graph is moving upward on the values 0.99 and 0.001 and also maintained a straight line. This shows that COVID 19 has the highest level in problem 2 than 1 because the heat has a significant effect on problem 2.

Thus, the result found in problem 2 shows that COVID 19 has the tendency of extension beyond the expected limit.

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