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On $\alpha - and\ \beta(\vartheta)$ Duals of the Double Sequence Spaces $\wp_\infty^2\ and\ \wp_{bc}^2$

Jidda Bashir[1] & Abubakar Masha[2]
[1]Department of General Studies, Mohammed Goni College of Legal, Islamic and Educational Studies, Maiduguri, Nigeria
[2]Department of Mathematics and Computer Science, Borno State University, Nigeria

Abstract*: This study introduces $\alpha - and\ \beta(\vartheta)$ duals of the double sequence spaces, $\wp_\infty^2\ and\ \wp_{bc}^2$. The properties of these introduced double sequence spaces were established, their $\alpha - and\ \beta - duals$ were also determined.*

Keywords: *Dual Space, Sequence Space, Pascal, Double Sequence, Matrix, Summability, Convergence.*

1. Introduction and Background

Başar and Sever (2009) introduced the space $\mathcal{L}_q$ of double sequences corresponding to the space ℓ_q of single sequences and examined some properties of the space $\mathcal{L}_q$. Furthermore, they determine the $\beta(\vartheta) - dual$ of the space and establish that the $\alpha -$ and $\gamma -$duals of the space $\mathcal{L}_q$ coincide with the $\beta(\vartheta) - dual$; where $1 \leq q < \infty$ and $\vartheta \in \{p, bp, r\}$. If P denotes the Pascal mean (a four dimensional matrix), then $\wp_\infty^2, \wp_c^2, \wp_{bc}^2$ and $\wp_0^2$ are collections of all double sequences whose P -transforms are in the spaces l_∞^2, c^2, c_b^2 and c_0^2 respectively; where l_∞^2, c^2, c_b^2 and c_0^2 are the double spaces of bounded, convergent, both bounded and convergent and null sequences respectively in the Pringsheim's sense, see Moricz (1991). We introduce a new double sequence space p_q^2 of Pascal as the set of all double sequences whose P -transforms are in the space l_q^2.

Karakaya and Polat (2010) introduced some new Para normed sequence spaces defined by Euler difference operators and computed their $\alpha -$, $\beta -$, and $\gamma -$duals. Also, some of the matrix transformations have been characterized. Karakaya et al., (2013) extended the sequence spaces which are defined in Karakaya and Polat, (2010) and Polat and Başar, (2007) by using difference operator of order m, and gave their $\alpha -$, $\beta -$, and $\gamma -$duals. Furthermore, they characterized some classes of the related matrix transformations. The new sequence spaces they defined are

$$e_0^r(\Delta^m, p) = \left\{ x = (x_k) \in \omega : \lim_{n\to\infty} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-r} r^k \Delta^m x_k \right|^{p_n} = 0 \right\} \quad (1)$$

$$e_c^r(\Delta^m,p)=\left\{x=(x_k)\in\omega:\lim_{n\to\infty}\left|\sum_{k=0}^{n}\binom{n}{k}(1-r)^{n-r}r^k(\Delta^m x_k-l)\right|^{p_n}=0, l\in\mathbb{R}\right\} \quad (2)$$

$$e_\infty^r(\Delta^m,p)=\left\{x=(x_k)\in\omega:\sup_n\left|\sum_{k=0}^{n}\binom{n}{k}(1-r)^{n-r}r^k\Delta^m x_k\right|^{p_n}<\infty\right\} \quad (3)$$

In 2018, Polat introduced the ordinary sequence spaces P_∞, P_c and P_0 , examining their properties and calculating their duals using matrix mappings. However, these spaces were not extended to double sequence spaces. Motivated by this gap, the present study focused on introducing double sequence spaces by extending P_∞, P_c and P_0 , naming them Pascal double sequence spaces, and exploring their properties. Additionally, the study defined a Pascal four-dimensional matrix and characterized matrix classes in a manner distinct from Polat's 2018 work.

The study is confined to concepts of double sequence spaces, properties of spaces and four-dimensional infinite matrices. The importance of the study is introduction of new summability method, new double sequence spaces and proofs of some properties of Pascal double sequence spaces.

2. Double Sequence Spaces

The following definitions would be found relevant to the study:

Definition 1 (Linty et al., 2019): Double sequence of real or complex numbers is a function defined on $N\times N$ whose range is the set of real or complex numbers. If $X: N\times N\to\mathbb{C}$, the image $X(n,m)$ of a positive integer n,m is usually denoted by $X_{n,m}$. For a fixed n, $(x_{n1},x_{n2},x_{n3},\dots,x_{nm}\dots)$ form the n^{th} row of the double sequence. For a fixed m, $(x_{1m},x_{2m},x_{3m},\dots,x_{nm},\dots)$ form the m^{th} column of the double sequence.

Definition 2 (Linty et al., 2019): Let $N\times N\to\mathbb{C}$ be a sequence of complex numbers let $X(n,m)$ be the double sequence represented by the equation $X(n,m)=\sum_{i=1}^{n}\sum_{j=1}^{m}a(i,j)$. The pair (a,x) is called the double series and it is represented by $\sum_{n,m=1}^{\infty}a_{n,m}$. Each number $a(n,m)$ is called the term of the series and $X(n,m)$ is known as partial sum.

Definition 3 (Bașar and Çolak, 2011): c_b^2 is the space of double sequences which are both convergent in the Pringsheim's sense and bounded, written $c_b^2=c^2\cap l_\infty^2$.

Definition 4 (Bașar and Çolak, 2011): A sequence in the space c^2 is said to be regularly convergent if it is a single convergent sequence with respect to each index and denote the set of all such sequences by c_r^2.

Definition 5 (Basar and Sever, 2009): A double sequence space X is solid if, and only if $\tilde{X}=\{u=(u_{jk})\in\omega^2:\exists\, x=(x_{jk})\in X \text{ such that} |u_{jk}|\le|x_{jk}|\, for\ all\ j,k\in\mathbb{N}\}\subset X$.

Definition 6 (Yeșilkayagil and Basar, 2016): The space X of double sequence spaces is monotone if $xu=(x_{jk}u_{jk})\in X$ for every $x=(x_{jk})\in X$ and $u=(u_{jk})\in\chi^2$, where χ^2 denotes the double sequence space of 0s and 1s.

Definition 7 (Bașar and Çolak, 2012): Let X,Y be two spaces of double sequences, with convergence rules $\vartheta_1-\lim$ and ϑ_2-lim, respectively, and $A=(a_{mnjk})$ also be a four-dimensional matrix over the real or complex field. Define the set $X_A^{\vartheta} = \Big\{x = (x_{jk}) \in \Omega: Ax = \big(\vartheta - \sum_{j,k} a_{mnjk}x_{jk}\big)_{m,n\in\mathbb{N}}\ exists\ and\ \in Y\Big\}$

then X_A^{ϑ} is called ϑ matrix domain of X and we say that A maps the space X into the space Y if $Y \subset X_A^{\vartheta}$. Denote the set of all four-dimensional matrices, mapping the space X into the space Y by $(X: Y)$. Then $(X: Y)$ is called matrix class. Then if any $A_{mnjk} \in (X: Y)$, then $(a_{mnjk})_{j,k}$ is in the $\beta(\vartheta)$ dual or β −dual represented by $X^{\beta(\vartheta)}$ of the space X for all $m, n \in \mathbb{N}$.

Definition 8 (Başar and Çolak, 2012): Let $\sum_{jk} a_{jk}x_{jk}$ be a double series, then the Abel's summation formula for double series is given by

$$\sum_{j,k=0}^{mn} a_{jk}x_{jk} = \sum_{j,k=0}^{m-1\,n-1} s_{jk}\Delta_{11}a_{jk} + \sum_{j=0}^{m-1} s_{jn}\Delta_{10}a_{jn} + \sum_{k=0}^{n-1} s_{mk}\Delta_{01}a_{mk} + s_{mn}a_{mn}\ \forall\, m,n \in \mathbb{N}$$

Definition 9 $(\textbf{Yeşilkayagil and Başar, 2015})$: The four dimensional summation matrix $S = (s_{mnkl})$ is defined by $s_{mnkl} \coloneqq \begin{cases} 1, & 0\le k\le m,\ 0\le l\le n, \\ 0, & \text{otherwise} \end{cases}$ for all $k, l, m, n \in \mathbb{N}$. Since S is a triangle, one can derive that the inverse S^{-1} is the four dimensional backward difference matrix $\Delta = (d_{mnkl})$ defined by $d_{mnkl} \coloneqq \begin{cases} (-1)^{m+n-(k+l)}, & 0\le k\le m,\ 0\le l\le n, \\ 0, & \text{otherwise} \end{cases}$ for all $k, l, m, n \in \mathbb{N}$. Define the sequence $y = (y_{mn})$ via the sequence $x = (x_{kl}) \in \Omega$ by $y_{mn} \coloneqq (Sx)_{mn} = \sum_{k,l=0}^{m,n} x_{kl}$ … (i) for all $m, n \in \mathbb{N}$. Let us suppose that the four dimensional matrices $A = (a_{mnkl})$ and $B = (b_{mnkl})$ transform the sequences $x = (x_{kl})$ and $y = (y_{kl})$ which are connected with (i) to the sequences $u = (u_{mn})$ and $v = (v_{mn})$, respectively, that is,

$u_{mn} = (Ax)_{mn} = \sum_{k,l=0}^{\infty} a_{mnkl}x_{kl}$ for all $\mathrm{m, n} \in \mathbb{N}$, … (ii)

$v_{mn} = (By)_{mn} = \sum_{k,l=0}^{\infty} b_{mnkl}y_{kl}$ for all $\mathrm{m, n} \in \mathbb{N}$. … (iii)

It is clear that the method B is applied to the $\mathrm{S}-$ transform of the sequence x, while the method A is directly applied to the elements of the sequence x. That is to say that the methods A and B are essentially different. Let us assume that the usual matrix product BS exists which is a much weaker assumption than the conditions on the matrix B belonging to any class of matrices, in general. We say in this situation that the matrices A and B in (ii) and (iii) are the *dual summability methods* if u is reduced to v and vice versa under the application of the formal summation by parts.

3. $\boldsymbol{\alpha - and\ \beta(\vartheta)}$ Duals of the Double Sequence Spaces $\boldsymbol{\wp_\infty^2\ and\ \wp_{bc}^2}$

We need the following general statements of **[Hamilton, 1936; Zeltser, 2002 and Zeltser, et al., 2009]** to calculate the $\alpha - and\ \beta(\vartheta)$ Duals of the Double Sequence Spaces $\wp_\infty^2\ and\ \wp_{bc}^2$.

Lemma 1: [Hamilton, 1936]: Let $\wp = (\wp_{mnjk})$ be a four-dimensional Pascal matrix, then $\wp \in (c_p, c_\vartheta)$ if and only if

$$\sup_{m,n\in\mathbb{N}} \sum_{j,k} |\wp_{mnjk}| < \infty \tag{3.1}$$

$$\exists\ \alpha_{jk} \in \mathbb{C} \ni\ \vartheta - \lim_{m,n} \wp_{mnjk} = \alpha_{jk}, \forall\, j,k \in \mathbb{N} \tag{3.2}$$

$$\exists\ \ell \in \mathbb{C} \ni\ \vartheta - \lim_{m,n\to\infty} \sum_{j,k} \wp_{mnjk} = \ell\ exists \tag{3.3}$$

$$\forall\, j \in \mathbb{N}, \exists\, k_0 \in \mathbb{N} \ni \ \wp_{mnjk} = 0 \; for\ every\ k > k_0\ and\ m, n \in \mathbb{N} \tag{3.4}$$

$$\forall\, k \in \mathbb{N}, \exists\, j_0 \in \mathbb{N} \ni \ \wp_{mnjk} = 0 \; for\ every\ j > j_0\ and\ m, n \in \mathbb{N} \tag{3.5}$$

Lemma 2: [Hamilton, 1936]: Let $\wp = (\wp_{mnjk})$ be a four-dimensional infinite matrix, then $\wp \in (c_{bp}: c_\vartheta)$ if and only if (3.1), (3.2) and (3.3) holds and,

$$\exists\, j_0 \in \mathbb{N} \ni \ \vartheta - \lim_{m,n\to\infty} \sum_k \left|\wp_{mnj_0k} - \wp_{j_0k}\right| = 0 \tag{3.2.1}$$

$$\exists\, k_0 \in \mathbb{N} \ni \ \vartheta - \lim_{m,n\to\infty} \sum_j \left|\wp_{mnjk_0} - \wp_{jk_0}\right| = 0 \tag{3.2.2}$$

In (3.2.2),

$x \in c_{bp}$ satisfies $\wp = (\wp_{jk}) \in \mathcal{L}_u$ and $\vartheta - \lim_{m,n\to\infty} [\wp x]_{mn} = \sum_{j,k} \wp_{jk} x_{jk} + (\ell - \sum_{jk} \wp_{jk})_{bp} - \lim_{m,n} x_{mn}$.

Lemma 3: [Cakan, et al., 2006]: Let $\wp = (\wp_{mnjk})$ be a four-dimensional infinite matrix, then $\wp \in (M_u: C_{bp})$ if and only if (3.1), (3.2) and (3.3)) holds and

$$\exists\, \wp_{jk} \in \mathbb{C} \ \ni \ bp - \lim_{m,n\to\infty} \sum_{j,k} \left|\wp_{mnjk} - \wp_{jk}\right| = 0 \tag{3.3.1}$$

$$bp - \lim_{m,n\to\infty} \sum_{k}^{n} \wp_{mnjk} \text{ exist for each } j \in \mathbb{N} \tag{3.3.2}$$

$$bp - \lim_{m,n\to\infty} \sum_{j}^{m} \wp_{mnjk} \text{ exist for each } k \in \mathbb{N} \tag{3.3.3}$$

$$\sum_{j,k} \left|\wp_{mnjk}\right| \ converges \tag{3.3.4}$$

In this section, $\alpha - and\ \beta(\vartheta)$ of the double sequences $\wp_\infty^2\ and\ \wp_{bc}^2$ are calculated.

Theorem 1: *Define the sets* $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$ *by*

$$\tau_1 = \left\{ x = (x_{jk}) \in \Omega \colon \begin{matrix} sup \\ m, n \end{matrix} \sum_{j,k} |\wp(m,n,j,k)| < \infty \right\} \tag{3.3.5}$$

$$\tau_2 = \left\{ x = \left(x_{jk}\right) \in \Omega \colon \exists\, \alpha_{jk} \in \mathbb{C} \ \ni \ \vartheta - \lim_{m,n\to\infty} \wp_{mnjk} = \alpha_{jk}, \forall j, k \in \mathbb{N} \right\} \tag{3.3.6}$$

$$\tau_3 = \left\{ x = \left(x_{jk}\right) \in \Omega \colon \exists\, \ell \in \mathbb{C} \ \ni \ \vartheta - \lim_{m,n\to\infty} \sum_{jk} \wp_{mnjk} = \ell\,, exists \right\} \tag{3.3.7}$$

$$\tau_4 = \left\{ x = \left(x_{jk}\right) \in \Omega \colon \exists\, j_0 \in \mathbb{N} \ \ni \ \vartheta - \lim_{m,n\to\infty} \sum_k \left|\wp_{mnj_0k} - \wp_{j_0k}\right| = 0 \right\} \tag{3.3.8}$$

$$\tau_5 = \left\{ x = (x_{jk}) \in \Omega : \exists\, k_0 \in \mathbb{N} \ni \vartheta - \lim_{m,n\to\infty} \sum_j \left| \wp_{mnj\ _0} - \wp_{jk_0} \right| = 0 \right\} \tag{3.3.9}$$

with

i. $(\wp_\infty^2)^\alpha = \tau_1$

ii. $(\wp_{bc}^2)^{\beta(\vartheta)} = \bigcap \tau_{i=1}^5$

Proof

i. To prove that $(\wp_\infty^2)^\alpha = \tau_1$. Let us take a double sequence $a = (a_{jk}) \in \tau_1$ and another double sequence $x = (x_{jk}) \in \wp_\infty^2$. Then there exists a double sequence $y = (y_{jk}) \in M_u$, while noting that

$$x_{jk} = \sum_{j,k}^{m,n} (-1)^{(m-j)+(n-k)} \binom{m}{m-j} \binom{n}{n-k} y_{jk} \tag{3.4.0}$$

with $m, n \in \mathbb{N}$, that there exists a real positive number$M > 0$ such that

$$\underset{j,k\in\mathbb{N}}{Sup} |y_{jk}| \le M \tag{3.4.1}$$

with these at hand, we have

$$\sum |a_{jk} x_{jk}| = \sum_{j,k} |a_{jk}| \left| \sum_{j,k}^{m,n} (-1)^{(m-j)+(n-k)} \binom{m}{m-j} \binom{n}{n-k} y_{jk} \right| \tag{3.4.2}$$

$$\sum |a_{jk} x_{jk}| \le M \sum_{j,k} |a_{jk}| \sum_{j,k=0}^{m,n} (-1)^{(m-j)+(n-k)}$$

$$\sum |a_{jk} x_{jk}| < \infty$$

This implies that $a = (a_{jk}) \in (\wp_\infty^2)^\alpha$ or $\tau_1 \subset (\wp_\infty^2)^2$ holds.

Conversely, suppose that $(a_{jk}) \in (\wp_\infty^2)^\alpha \backslash \tau_1$. Then, we have $\sum_{j,k} |a_{jk} x_{jk}| < \infty$ for all $x = (x_{jk}) \in \wp_\infty^2$. This is clear if we take $x = (x_{jk}) = ((-1)^{m-j}) \in \wp_\infty^2$ that $\sum_{j,k} |a_{jk} x_j| = \sum_{j,k} |a_{jk}| = \infty$. This means $(a_{jk}) \notin (\wp_\infty^2)^\alpha$. This is a clear contradiction to our assumption. So $(a_{jk}) \in \tau_1$; or $(\wp_\infty^2)^\alpha \subset \tau_1$. This proves the theorem.

ii. Let $a = (a_{jk}) \in \omega^2$and $x \in (\wp_{bp}^2)$. Then $y = \wp x \in C_{bp}$, and we have by $(m,n)^{th}$ partial sum that

$$\sum_{j,k}^{m,n} a_{jk} x_{jk} = \sum_{j,k=0}^{m,n} a_{jk} \sum_{r,s} (-1)^{(m-r)+(n-s)} \binom{m}{m-r} \binom{n}{n-s} y_{rs} \tag{3.4.3}$$

$$\sum_{j,k}^{m,n} a_{jk}x_{jk} = \sum_{j,k=0}^{m,n}\left[\sum_{r,s=j,k}^{m,n}(-1)^{(r-m)(s-n)}\binom{m}{m-r}\binom{n}{n-s}a_{rs}\right]y_{k,l} \qquad *$$

$$\sum_{j,k}^{m,n} a_{jk}x_{jk} = Dy_{m,n}\text{ , say.} \tag{3.4.4}$$

where D is defined by $D = (d_{jk}^{mn})$ is defined by

$$d_{jk}^{mn} = \begin{cases}\sum_{r,s=j,k}^{m,n}(-1)^{(r-m)(s-n)}\binom{m}{m-r}\binom{n}{n-s}a_{rs}\text{ , } for\ 0 \le k \le m\ and\ 0 \le l \le n \\ 0, \qquad elsewhere, \forall\ j,k,m,n \in \mathbb{N}\end{cases} \tag{3.4.5}$$

Since (*) holds, one can conclude that $ax \in CS_\vartheta$, where $x = (x_{jk}) \in \wp_{bc}^2$ if and only if $Dy \in C_\vartheta$ whenever $y = (y_{jk}) \in C_{bp}$. This allows $a = (a_{jk}) \in {\wp_{bc}^2}^{\beta(\vartheta)}$ if and only if $D \in (C_{bp}, C_\vartheta)$. Hence the condition $\tau_{i=1}^5$ of the theorem holds.

$$\tau_1 = \left\{x = (x_{jk}) \in \Omega\text{: } \sup_{\mathrm{m,n}}\sum_{\mathrm{j,k}}|\wp(\mathrm{m,n,j,k})| < \infty\right\} \tag{3.4.6}$$

$$\tau_2 = \left\{x = (x_{jk}) \in \Omega\text{: } \exists\ \alpha_{jk} \in \mathbb{C}\ \ni\ \vartheta - \lim_{m,n\to\infty}\wp_{mnjk} = \alpha_{jk}, \forall j,k \in \mathbb{N}\right\} \tag{3.4.7}$$

$$\tau_3 = \left\{x = (x_{jk}) \in \Omega\text{: } \exists\ \ell \in \mathbb{C}\ \ni\ \vartheta - \lim_{m,n\to\infty}\sum_{jk}\wp_{mnjk} = \ell\text{ , } exists\right\} \tag{3.4.8}$$

$$\tau_4 = \left\{x = (x_{jk}) \in \Omega\text{: } \exists\ j_0 \in \mathbb{N}\ \ni\ \vartheta - \lim_{m,n\to\infty}\sum_{k}\left|\wp_{mnj_0k} - \wp_{j_0k}\right| = 0\right\} \tag{3.4.9}$$

$$\tau_5 = \left\{x = (x_{jk}) \in \Omega\text{: } \exists\ k_0 \in \mathbb{N}\ \ni\ \vartheta - \lim_{m,n\to\infty}\sum_{j}\left|\wp_{mnjk_0} - \wp_{jk_0}\right| = 0\right\} \tag{3.5.0}$$

Conclusion

The study successfully introduces the $\boldsymbol{\alpha - and\ \beta(\vartheta)}$ -duals of the double sequence spaces $\boldsymbol{\wp_\infty^2\ and\ \wp_{bc}^2}$, extending concepts from traditional sequence spaces into the realm of double sequences. By establishing the properties and providing explicit formulas for these duals, the research contributed to a deeper understanding of the relationships between various types of sequence spaces and their duals. The use of Pascal matrices and the application of specific convergence rules further enriches the mathematical structure of these spaces. Additionally, the conditions and proofs presented in the study offer valuable insights for future work in the field of summability theory and the study of matrix transformations. The established theorems and lemmas serve as foundational tools for further exploration of dual spaces and their applications to different types of sequences, particularly in the context of Pascal matrix transformations.

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