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ABSORBING MARKOV CHAIN STOCHASTIC MODEL OF STUDENTS' GRADUATING GRADES

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Abstract: *Getting admission into Nigeria tertiary institutions and graduating within stipulated time have become the two sides of the hydra-headed coin of tertiary education in Nigeria which requires scientific digitalization and statistical research approach to solve such problems. The use of grade point average, grade level and graduating grades for the purposes of academic level progression, graduation standards, academic amnesty and various academic Honours in the Nigeria Polytechnic National Diploma programme is of immense statistical research interest. So, this paper seeks to determine the probability of a student that starts with a grade level in the first semester will withdraw from the programme or graduate within the first four semesters of the programme and to determine the average number of semesters a student that starts with any of the grade levels in first semester will spend before graduating from the programme. Data on the grade point average of a sample of 233 students in a department of a Nigerian Polytechnic for four semesters of two academic sessions of the National Diploma programme were collected and used for this study. The absorbing Markov chain model consisting of eight states (two absorbing states and six transient states) was fit to the data. The withdrawal and graduation probabilities were obtained. Also, the average number of semesters a student that starts with any of the grade levels in the first semester result will spend before graduating from the programme is determined. The data was analysed using STATA and MATLAB softwares.*

Keywords: *Grade Point Average, Graduating grades, Grade level, Absorbing Markov Chain.*

I. INTRODUCTION

Getting admission into tertiary institutions in Nigeria and graduating within stipulated time have become the two sides of the hydra-headed coin of the tertiary education in Nigeria which requires scientific digitalization and statistical research approach to solve such problems. Every student seeking admission into any tertiary institution in Nigeria be it Monotechnic, College of Education, College of Health Sciences, Polytechnic or University is faced with this challenge of how to graduate with good grade and on time too. This challenge cuts across all tertiary institutions the world over, be it private, State or Federal owned institutions, (Pierre & Silver, 2016; Idiata Edoimioya & Aigbomian, 2008); though the magnitude could differ slightly across institutions.

The National Policy of Education of the Federal Republic of Nigeria (2013) includes the Polytechnic Education as part of the fourth level of the 6:3:3:4 system of education in Nigeria. As the mainstream of the Technical Tertiary Education, the Polytechnic education has its aims and objectives directed toward providing applied scientific, technological and commerce knowledge and skills necessary for agricultural, industrial and economic development. Amaghionyeodiwe and Osinubi (2006). The National Board for Technical Education (NBTE) (2001, 2014) guidelines for the establishment of private Polytechnics, Monotechnic and similar tertiary institutions in Nigeria has it that a Polytechnic is a technical institution offering post-secondary technical education programmes leading to the award of diplomas/certificates such as the National Diploma (ND) and Higher National Diploma (HND). The products of these institutions will have entry level employment skills to function as technicians, higher technicians/technologists or professionals, depending on the level of training, in their fields of specialization. A polytechnic may also be involved in applied research, and perform any other functions that its council may require it to perform from time to time. Adefunke and Olaitan (2020); Ovbiagele (2015) and Idoko (2005) pointed out that technical education obtainable at the Polytechnic is clearly distinguishable from literacy education.

Among the unique characteristics of the Polytechnic education in Nigeria is its grading system and graduating grades. Omotosho (2013) identified that the cumulative grade point average, (CGPA) is the assessment instrument adopted in most tertiary institutions around the world because of its unique features. The use of grade point average (GPA), grade level and graduating grades for the purposes of academic level progression, graduation standards, academic amnesty and various academic Honours in the Nigeria Polytechnic National Diploma programme is of immense statistical research interest. The student handbooks of several Nigeria Polytechnics surveyed showed that students that made less than 1.50 grade point average (GPA) will be required to withdraw from the programme, students that made between 1.50 and 1.99 grade point average will be required to repeat the programme by starting the program all over, students that made between 2.00 and 2.49 grade point average will be graded at pass level and are to continue the programme, students that made between 2.50 and 2.99 grade point average will be graded as lower credit and such are expected finish well in the programme, students that made between 3.00 and 3.49 grade point average will be graded as upper credit and such have higher chance of ending better at the end of the programme and students that made between 3.50 and 4.00 grade point average will be graded as distinction which is the highest grade of the programme. The introduction of withdrawal condition ensures that the rest of the students that continue with the programme will graduate with any of these levels, pass, lower credit, upper credit or distinction. This paper seeks to apply the absorbing chain stochastic model in answering the questions of students' graduation and withdrawal in the National Diploma programme of Polytechnic education in Nigeria.

Therefore, the objectives of this research paper are to determine the probability of a student that starts with any of the grade levels in the first semester result will withdraw from or graduate within the stipulated first four semesters of the National Diploma programme and to determine the average number of semesters a student that starts with

any of the grade levels in the first semester result will spend before graduating from the National Diploma programme. These objectives led to these formulated research questions: what is the probability that a student that starts with any of the grade levels in the first semester result will withdraw from or graduate within the stipulated first four semesters of the National Diploma programme and what is the average number of semesters a student that starts with any of the grade levels in the first semester result will spend before graduating from the National Diploma programme. This research work will be of great benefits in the following ways: (1) institutions of higher learning with similar model condition will use the model for the appraisal of their various departments in determining the department that has better chances of earlier graduation of their Diploma students. (2) students seeking admissions into higher institutions will be properly guided in their choice of institutions and departments so as to meet up with their expectation of graduating at a desired time frame. (3) the findings of this study will serve as early warning intervention and monitoring system that reduces the percentage of students with risk indicators related to low GPA in the first semester of the National Diploma Programme.

II. LITERATURE REVIEW

Shah and Burke (1999) used the Markov Chain to model the flow of undergraduate students of higher education system of Australia and tried to find out the probability completion, mean time takes to complete the course, and the time they spend in their study. Alawadhi and Konsowa (2007) used Markov chain to study student flow in a higher educational institution. They used historical data of a random sample of 250 students of the faculty of science at Kuwait university and the result show that: (i) a freshman student has about 0.401 probability of graduating, (ii) freshman, sophormore, junior stay on average 3, 2.3, 2.2 semesters at their respective levels before they pass on to the next level of study, while senior students stay longer, an average of 3.7 semesters, (iii) a high percentage of 39% of incoming freshman student withdraw from their study, and (iv) the probability of progression to a higher level and graduating increases as students move on to a higher level in the system.

Musiga, Owino and Weke (2011) in their paper “Modeling a hierarchical system with double absorbing States” describes a Markov Chain transition model for predicting expected numbers of successful graduates and unsuccessful dropouts from an education system. They used double absorbing states, improvement on their single absorbing state model where successful and unsuccessful dropouts were grouped together. The double absorbing states model enables them to predict the numbers of expected qualified personnel vis-à-vis the numbers of unsuccessful dropouts from a system. Pierre and Silver (2016) studied student retention using graduate operations research class to determine predictors of the length of time it took a student to graduate from Clark Atlanta University. They use Markov chains to analyse the behavior of student enrollment in discrete-time as students transition from one discrete state space classification (Freshman, Sophomore, Junior and Senior) to the next. The study found out that more than 95% of students with a B average or better in their freshman year graduate, Freshmen who earn a B average or better have an 80 percent chance of finishing high school with at least a 3.0 GPA, Freshmen with less than a C average are more likely to drop out than graduate, those with GPAs of C- or D who miss one or two weeks of school

per semester. These middle students have about the same chances of graduating as they have of dropping out. Hlavatý and Dömeová (2014) considered students' progress throughout examination process in economics at the Czech university of life sciences, Prague, Czech Republic. They created a model of students' progress throughout the whole course using the Markov chain approach. Each student has to go through various stages of the course requirements where his success depends on the completion of the previous duties. The absorbing states they considered are failed, good, very good and excellent while the transient states are exam enrolled, practical part successfully finished, entire test finished (50-59 points), entire test finished (60-100/120 points). Two groups of students were observed separately depending whether they have some bonus points (175 students) or not (103 students). They show how students' achievements during the semester affect the final result of the exam in terms of the final grade as the final grade was shown to be an absorbing state of a Markov chain that is terminal and cannot be transitioned from. With the help of Markov chains, the probability of success at the end of the course is shown regarding the students' behaviour and diligence during the course. The results of their study show that in the first stage, the expected number of steps (moves from one state to another) is 2.52 in average. Of course, with the increasing number of steps taken, the number of expected steps before absorption lowers. If one is in the state at failed 2nd attempt the expected number of remaining steps is obviously 1 as there is no other option of more steps to be taken. In the second stage of the experiment having students without and with bonus points, the results show that the students with bonus points from the semester, i.e. those working better continuously, have better chance to pass the exam. There are no big differences between these two groups of students in the probability to fail the exam after being successful in the examination test: compare the values 0.1 and 0.11. They found that students who are able to write the test with satisfactory number of points can pass the exam without the bonus points and hence conclude that partial evaluation during the semester is more helpful to the weaker students, an inverse effect comparing with the higher grading standards.

Brezavšček, Bach & Baggia (2017) researched on students' progression towards completing their higher education degrees in Slovenia using absorbing Markov chain. They develop a stochastic model for estimation and continuous monitoring of various quality and effectiveness indicators of a given higher education study programme having five transient states (1 – the student is enrolled into the first year of the study programme, 2 – the student is enrolled into the second year of the study programme, 3 – the student is enrolled into the third year of the study programme, C – the student is enrolled into the candidate year and I – the student is currently inactive) and two absorbing states (G – the student has graduated and successfully finished the study programme and W – the student has withdrawn from the study programme). The model is applied to investigate the pattern of students' enrolment and their academic performance in a Slovenian higher education institution. Based on the students' intake records, the transition matrix was developed considering eight consecutive academic seasons from 2008/09 until 2016/17. The students' progression towards the next stage of the study programme was estimated as: fraction of students who successfully progress from the first to the second year is 0.31, fraction of students who successfully progress from the second to the third year is 0528

and fraction of students who successfully progress from the third to the candidate year or to graduation is 0.891. The expected time that a student spends at a particular stage as well as the expected duration of the study is determined to be 5.572, 5.29, 5.39, 5.401, 5.317, 5.358 and 5.395 respectively from the first-year of 2010/2011 academic year to the 2016/2017. The graduation probabilities were obtained to be 0.20, 0.55, 0.82, 0.85 and 0.04 respectively from the first-year of 2010/2011 academic year to the 2016/2017. The withdrawal probabilities were obtained to be 0.80, 0.45, 0.18, 0.15 and 0.96 respectively from the first-year of 2010/2011 academic year to the 2016/2017. Also, they used their model to make a prediction on the students' enrolment for the next three academic years.

Adams, Abdulkadir and Jibasen (2019) study the pattern of students' movement within and around the various classes of degrees in Modibbo Adama University of Technology, Yola, Nigeria. In their paper, they developed a transition matrix for the five classes of degrees using movement patterns in ten consecutive semesters (2011 – 2016). The probabilities of moving across the five different classes were obtained. The model was applied to data collected on academic performance, captured by the current Grade Point Average (GPA) of students who enrolled and graduated within the period 2011 – 2016, from seven departments, one each from seven schools of the University. The Department are; Statistics and Operations Research, Management Technology, Agric. Economics, Mathematics Education, Biochemistry, Architecture and Civil Engineering. The data collected on GPA was coded as follows; first class, second class upper, second class lower, third class and pass. This gives five classes of degree defined as states of the process. The students' results were collected per semester, and a maximum of nine semesters were recorded, given rise to a maximum of eight trials per student. The analysis results on Individual-Level Transition Matrix show that, 20.8% students moved from first class to second class upper, 19.1% from first class to second class lower, 28.5% from third class to second class lower, 21.9% from pass degree to second class lower as well as 22.4% from pass degree to third class grade which is more when compared to other grades, with third class to second class lower recording the highest movements, while pass degree to first class and first class to pass degree recording the lowest movements. The results on the Stayer Population on Academic Performance shows that, 2.0% of the students who started with first class as their baseline grade retained that grade throughout the period of study; 2.7% started with second class upper and remained there through the entire study period. Similarly, 3.6%, 2.4% and 1.6% of students who started as second class lower, third class and pass degree respectively retained the grades throughout the period of study. Yakubu, Shamsuddeen, Ibrahim & Ibrahim (2020) modelled student's academic performance and progression in Indian Private Universities using stochastic Markov Chain to study the pattern of student's movement across various study levels at department of Business Administration Mewar University. Their model has three transient and two absorbing states. The results of their model showed that, a student at Mewar University is expected to spend 1.0270 academic year for the first year, 0.9411 academic year for the second year and 0.9060 academic year for the third year.

Shahab and Adan (2020) apply a sensitivity analysis to compare the performance of American standard six-year graduation rate method with that of absorbing Markov chains.

They propose and the use of a regularly updating multi-level absorbing Markov chain (RUML-AMC) in which the transition matrix is updated year to year. They empirically demonstrate that the proposed RUML-AMC approach nearly eliminates estimation bias while reducing the estimation variation by more than 40%, especially for populations with small sample sizes. Ezugwu and Ologun (2017) used the Markov Chain as a predictive model for manpower planning. The movements of staff they call transitions and sought to determine the proportions of staff recruited, promoted and withdrawn from the various grades and to forecast the academic staff structure of university of Uyo in the next five years. Their results showed that there is a steady increase in the number of Graduate Assistants, Senior Lecturer and Associate professors, while, there is a steady decrease in the number of Assistant Lecturer, Lecturer II, Lecturer I, and Professor in the next five years. Aparna and Sarat (2017) applied Absorbing Markov chain in the analysis of students' performance of Nagaon District under Gauhati University. The results of six degree offering colleges of the district were analysed and found the average time students spent in each semester, the number of semesters required to become a graduate and the probability of students' graduation and withdrawal from the course.

Odukoya, Bowale & Okunlola (2018) review the formulation and implementation of the Nigerian national policy of education that was enacted in 1977. Though the document featured lot of laudable policies, feedback from key stakeholders in education revealed that the implementation of the policy was deficient and need urgent attention for the country to experience the much need development. Murtala, Jamilu & Usman (2019) studies the pattern of students' enrolment and their academic performance in the department of Statistics Ahmadu Bello University, Zaria. They developed a stochastic Markov chain with 4 transient and two absorbing states. The probability transition matrix was constructed. The students' progression towards the next stage of the study programme was estimated, the expected time that a student spends at a particular stage as well as the expected duration of the study determined. The probation, withdrawal and graduation probabilities were also obtained. They analyzed their data using R- Software. Mavruk and Ersin (2016) studies the academic progress of vocational school of social sciences students in quantitative courses in order to improve student progress in education using stochastic Markov chain model. A population of 232 students taking commercial mathematics and statistics courses in 2014/2015 academic year is selected. The results of their analysis showed that in 2028-2029 academic year, the probabilities of improvement, no change and decline become stable at 28%, 20% and 52%, respectively. Ledwith (2019) in his unpublished thesis studies the application of absorbing Markov chain to assess education attainment rates within Air Force Materiel Command Civilian Personnel to forecast the educational composition of the institution workforce to enable organizational performance improvements. The results of the analysis indicate that the four combinations of stationarity assumptions perform similarly at representing the historical data and the forecasted educational attainment rates of the institution workforce are expected to increase significantly.

The literature reviewed show that an absorbing Markov chain model of a Nigerian polytechnic education grading system with the peculiarities explained above has not been researched on. So, this research work intends to fill this gap.

III. METHODOLOGY

TRANSITION MATRIX

Markov chain is a discrete time stochastic process whose past states only depends on the present state, Kemeny and Snell (1967) and Bartholomew (1973). This is expressed as

$$P(X_{n+1} = j/\, X_0 = x_0, \dots, x_{n-1}, X_n = i) = P(X_{n+1} = j/\, X_n = i) = P_{ij} \qquad (1)$$

P_{ij} is the transition matrix consisting of probabilities of moving from one state to another state.

$$P_{ij} = \begin{pmatrix} P_{11} & P_{12} & . & . & . & P_{1k} \\ P_{21} & P_{22} & . & . & . & P_{2k} \\ \cdot & \cdot & . & . & . & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{k1} & P_{k2} & . & . & . & P_{kk} \end{pmatrix} \qquad (2)$$

where the finite set $I_k = \{1, 2, \dots, k\}$ is called the state space of the Markov chain. The entries of P_{ij} in the transition matrix satisfies these conditions:$(i)\ P_{ij} \geq 0\ (ii) \sum_j P_{ij} = 1\ and\ i, j \in I_k$

THEORY OF ABSORBING MARKOV CHAIN

The theory of absorbing Markov chain with finite states having r absorbing and t transient states is given by the transition matrix P_{ij} in the following canonical general form:

$$P_{ij} = \boldsymbol{P} = \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} = \begin{bmatrix} \begin{bmatrix} Q_{11} & \cdots & Q_{1t} \\ \vdots & \ddots & \vdots \\ Q_{t1} & \cdots & Q_{tt} \end{bmatrix} & \begin{bmatrix} R_{11} & \cdots & R_{1r} \\ \vdots & \ddots & \vdots \\ R_{t1} & \cdots & R_{tr} \end{bmatrix} \\ \begin{bmatrix} O_{11} & \cdots & O_{1t} \\ \vdots & \ddots & \vdots \\ O_{r1} & \cdots & O_{rt} \end{bmatrix} & \begin{bmatrix} I_{11} & \cdots & I_{r1} \\ \vdots & \ddots & \vdots \\ I_{t1} & \cdots & I_{rr} \end{bmatrix} \end{bmatrix} \qquad (3)$$

where $\boldsymbol{Q}$ is $t \times t$ matrix expressing transitions between transient states,

$\boldsymbol{R}$ is nonzero matrix $t \times r$ expressing transitions from transient states to absorbing states,

O is the $r \times t$ zero matrix and

$\boldsymbol{I}$ is identity matrix of order $r \times r$.

FUNDAMENTAL MATRIX

The fundamental matrix $\boldsymbol{N}$ is given as

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \qquad (4)$$

where $\boldsymbol{I}$ in (4) is a $t \times t$ matrix and is different from the matrix $\boldsymbol{I}$ of (3) which is $r \times r$.

ABSORPTION PROBABILITY

Let b_{ij} be the probability that the absorbing chain will be absorbed in an absorbing state j when it starts from transient state i. Let $\boldsymbol{B}$ be matrix with components b_{ij}. Then $\boldsymbol{B}$ is an $t \times r$ matrix:

$$\boldsymbol{B} = \boldsymbol{NR} \qquad (5)$$

where $\boldsymbol{N}$ is the fundamental matrix and $\boldsymbol{R}$ is the sub-matrix from the transition matrix $\boldsymbol{P}$ above.

TIME OF ABSORPTION

Elements of the matrix $\boldsymbol{N}$ express how many times the Markov process in average reaches the transient states. The expected number of steps before the chain is absorbed is the time of absorption, $\boldsymbol{\mu}$ given as

$$\boldsymbol{\mu} = \boldsymbol{Nc} \tag{6}$$

where $\boldsymbol{N}$ is the fundamental matrix and **c** is a column vector of all entries equal to 1.

MODEL

The P_{ij} given in (2) in this research paper is the transition matrix consisting of probabilities associated with student moving from one grade level to another as they transit from one semester to another in the four semesters of the ND programme. The two absorbing states are Withdraw and Graduate since every state (grade level) can reach any of these two absorbing states of and it is impossible to leave this state once it is reached. The transients' states identified are the grade levels that the students can transit among them as given below:

Grade level	Grade point average (GPA)
Inactive	less than 1.50
Repeat	between 1.50 and 1.99
Pass	between 2.00 and 2.49
Lower credit	between 2.50 and 2.99
Upper credit	between 3.00 and 3.49
Distinction	between 3.50 and 4.00

The ND programme of Polytechnic education in Nigeria and particularly the institution of study as at the time the data was collected is in line with the following states:

- Inactive – the student is enrolled into the first year of the ND programme, at the end of the first semester could not make up to 1.50 grade point average and is termed inactive but continued in the programme.
- Repeat – the student at the end of the first semester made between 1.50 and 1.99 grade point average and is considered to repeat but continued in the programme.
- Pass – the student at the end of the first semester made between 2.00 and 2.49 grade point average considered pass and continued in the programme.
- Lower credit – the student at the end of the first semester made between 2.50 and 2.99 grade point average considered Lower credit and continued in the programme.
- Upper credit – the student at the end of the first semester made between 3.00 and 3.49 grade point average considered Upper credit and continued in the programme.
- Distinction – the student at the end of the first semester made between 3.50 and 4.00 grade point average considered Distinction and continued in the programme.
- Withdraw – the student withdraws from the programme. And a student in any of the six transient states can withdraw from the programme.
- Graduate – the student graduates and successfully finishes the ND programme.

ASSUMPTIONS

In developing the model, the following assumptions are considered:

1. The student who is currently enrolled into the first semester of the study programme can, in the following semester, either progress to a higher grade level state or stay at the same state of grade level.
2. The student who is in inactive state could withdraw from the programme or continue the programme.
3. The student who has withdrawn will never finish the programme.

IV. RESULTS AND DISCUSSSION

Data on grade point average was collected on a sample of 233 students of a department in a Nigerian Polytechnic for the four semesters across the two academic sessions. The Markov chain states for this research paper consists of eight states of which two states are absorbing states {Withdraw (W), Graduate, G} and six states are transients state {Inactive, Repeat, Pass, Lower credit, Upper credit, Distinction}. As at the time of collecting the data 2012 and 2013, the Nigeria polytechnic grading system and credit units in bracket are as following: 80-100 is A (4), 70-79 is AB (3.5), 60-69 is B (3), 50-59 is BC (2.5), 40-49 is C (2.0), 30-99 is CD (1.5), 20-29 is D (1.0), 10-19 is E (0.5), and 0-9 is F (0). These are used to calculate the grade point average (GPA) at the end of each of the four semesters period. And the data were entered into STATA as a panel dataset and the transition matrix obtained are shown below:

$$P_{ij} = \begin{array}{c} \begin{matrix} GPA & \text{Inactive} & Repeat & Pass & LowerCredit & \text{UpperCredit} & Distinction & Withdraw & Graduate \end{matrix} \\ \begin{pmatrix} \text{Inactive} & 0.4878 & 0.0488 & 0.0487 & 0.0244 & 0 & 0 & 0.3415 & 0.0488 \\ Repeat & 0.1125 & 0.2125 & 0.4000 & 0.2000 & 0 & 0 & 0.0250 & 0.0500 \\ Pass & 0.0269 & 0.1228 & 0.4701 & 0.3054 & 0.0449 & 0 & 0.0030 & 0.0269 \\ LowerCredit & 0 & 0.0075 & 0.2180 & 0.4023 & 0.1316 & 0.0038 & 0.0075 & 0.2293 \\ \text{UpperCredit} & 0 & 0.0093 & 0.0187 & 0.2056 & 0.3645 & 0.0468 & 0.0187 & 0.3364 \\ Distinction & 0 & 0 & 0 & 0 & 0.1111 & 0.3333 & 0 & 0.5556 \\ Withdraw & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ Graduate & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Decomposing the transition matrix P_{ij} above into the transient state (Q_{ij}), absorbing state (R_{ij}), an identity matrix (I_{ij}) and the zero matrix (O_{ij}) respectively are as follows:

$$\boldsymbol{Q} = Q_{ij} = \begin{pmatrix} 0.4878 & 0.0488 & 0.0487 & 0.0244 & 0 & 0 \\ 0.1125 & 0.2125 & 0.4000 & 0.2000 & 0 & 0 \\ 0.0269 & 0.1228 & 0.4701 & 0.3054 & 0.0449 & 0 \\ 0 & 0.0075 & 0.2180 & 0.4023 & 0.1316 & 0.0038 \\ 0 & 0.0093 & 0.0187 & 0.2056 & 0.3645 & 0.0468 \\ 0 & 0 & 0 & 0 & 0.1111 & 0.3333 \end{pmatrix}$$

$$\boldsymbol{R} = R_{ij} = \begin{pmatrix} 0.3415 & 0.0488 \\ 0.0250 & 0.0500 \\ 0.0030 & 0.0269 \\ 0.0075 & 0.2293 \\ 0.0187 & 0.3364 \\ 0 & 0.5556 \end{pmatrix}$$

$$\boldsymbol{O} = O_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrix $\boldsymbol{I}$ of (3) which is $r \times r$ i.e. 2×2 in this case is

$$\boldsymbol{I} = I_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Fundamental Matrix Computation

The fundamental matrix $\boldsymbol{N}$ is calculated using $\boldsymbol{I}$ in (4) is a $t \times t$ matrix, i.e. 6×6 matrix:

$$\boldsymbol{I} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{I} - \boldsymbol{Q} = \begin{pmatrix} 0.5122 & -0.0488 & -0.0487 & -0.0244 & 0 & 0 \\ -0.1125 & 0.7875 & -0.4000 & -0.2000 & 0 & 0 \\ -0.0269 & -0.1228 & 0.5299 & -0.3054 & -0.0449 & 0 \\ 0 & -0.0075 & -0.2180 & 0.5977 & -0.1316 & -0.0038 \\ 0 & -0.0093 & -0.0187 & -0.2056 & 0.6355 & -0.0468 \\ 0 & 0 & 0 & 0 & -0.1111 & 0.6667 \end{pmatrix}$$

The MATLAB software was used to obtain $\boldsymbol{N}$, $\boldsymbol{B}$ and $\boldsymbol{\mu}$ given below.

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} = \begin{pmatrix} 2.0299 & 0.2194 & 0.5595 & 0.4915 & 0.1436 & 0.0129 \\ 0.4668 & 1.6390 & 2.0332 & 1.7858 & 0.5216 & 0.0468 \\ 0.2867 & 0.5425 & 3.1897 & 2.0500 & 0.6600 & 0.0468 \\ 0.1227 & 0.2451 & 1.3116 & 2.6550 & 0.6531 & 0.0610 \\ 0.0557 & 0.1207 & 0.5548 & 0.9571 & 1.8344 & 0.1342 \\ 0.0093 & 0.0201 & 0.0924 & 0.1595 & 0.3057 & 1.5223 \end{pmatrix}$$

Computation of Probability of Absorption (Withdraw or Graduation)

To determine the probability of a student that starts with any of the grade levels in his/her first semester result will withdraw or graduate within the stipulated first four semesters of the National Diploma programme, the matrix $\boldsymbol{B}$ is computed as

$$\boldsymbol{B} = \boldsymbol{NR} = \begin{pmatrix} 0.7068 & 0.2932 \\ 0.2296 & 0.7704 \\ 0.1487 & 0.8513 \\ 0.0841 & 0.9159 \\ 0.0652 & 0.9348 \\ 0.0109 & 0.9891 \end{pmatrix}$$

The first column of the matrix $\boldsymbol{B}$ is the probability of withdrawing while the second column of the matrix $\boldsymbol{B}$ is the probability of Graduation.

To calculate the time until Absorption (Withdraw or Graduation), that is, to determine the average number of semesters a student that starts with any of the grade levels in his/her first semester result will spend before graduating from the National Diploma programme; the vector $\boldsymbol{\mu}$ is computed and it is

$$\boldsymbol{\mu} = \boldsymbol{Nc} = \begin{pmatrix} 2.0299 & 0.2194 & 0.5595 & 0.4915 & 0.1436 & 0.0129 \\ 0.4668 & 1.6390 & 2.0332 & 1.7858 & 0.5216 & 0.0468 \\ 0.2867 & 0.5425 & 3.1897 & 2.0500 & 0.6600 & 0.0468 \\ 0.1227 & 0.2451 & 1.3116 & 2.6550 & 0.6531 & 0.0610 \\ 0.0557 & 0.1207 & 0.5548 & 0.9571 & 1.8344 & 0.1342 \\ 0.0093 & 0.0201 & 0.0924 & 0.1595 & 0.3057 & 1.5223 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.4568 \\ 6.4932 \\ 6.7757 \\ 5.0485 \\ 3.6569 \\ 2.1093 \end{pmatrix}$$

Discussion of Results

From the matrix of probability of absorption given as $\boldsymbol{B}$ in this paper**,** the results show that the probability of withdrawal decreases as the grade level of the students moves from the lowest level of Inactive (less than 1.50), Repeat (between 1.50 and 1.99), Pass (between 2.00 and 2.49), Lower credit (between 2.50 and 2.99), Upper credit (between 3.00 and 3.49) and to the highest grade level of Distinction (between 3.50-4.00) at this rate 0.7068, 0.2296, 0.1487, 0.0841, 0.0652, 0.0109 respectively while the probability of graduation increases for each of the grade level as 0.2932, 0.7704, 0.8513, 0.9159, 0.9348 and 0.9891 respectively. These results are in tandem with the a priori expectation of an academic programme.

The expected time for the student to withdraw or graduate given in the vector $\boldsymbol{\mu}$ shows that it takes an average number of 3.4568 approximately 3 semesters for a student that starts with the inactive grade level to withdraw from the programme, a student that starts with repeat grade level will spend about 6.4932 approximately 6 semesters and end up withdrawing from that National Diploma programme; but a student that starts with pass grade level will spend about 6.7757 approximately 7 semesters and end up graduating from that National Diploma programme, a student that starts with lower credit grade level will spend about 5.0485 approximately 5 semesters and end up before graduating from that National Diploma programme; a student that starts with upper credit grade level will spend about 3.6569 approximately 4 semesters to graduating from the National Diploma programme and a student that starts with distinction grade level will spend about 2.1093 approximately 2 semesters to graduating from the National Diploma programme.

V. CONCLUSION

The absorbing Markov chain stochastic model has been developed in this paper to determine the graduation and withdrawal progression of students in a department of a Nigerian Polytechnic using their grade point averages and cumulative grade averages. This model shows the appropriate movement of probability: decreasing probability of withdrawal from the first semester grade of inactive, repeat, pass, lower credit, upper credit and to the highest grade level of distinction while the probability of graduation increases for each of the grade level. The model also determined the expected time for the student to withdraw or graduate which shows the time that it takes for a student that

starts with each of grade level to withdraw or graduate from the programme. The expected time for graduation of distinction grade level students given as approximately two semesters is a pointer to the fact that the policy of pegging the diploma and degree programmes in Nigeria education should be revisited as students should be allowed the latitude to graduate from programmes at their individual pace.

REFERENCES

Adams Y.J., Abdulkadir S.S. & Jibasen D. (2019). *Mover-Stayer Analysis of Students' Academic Progress in Modibbo Adama University of Technology, Yola. Research Journal of Education. 5*(9), 161-166.

Adefunke T.J. & Olaitan O.F. (2020). *Critique of the National Policy on Technical, Vocational Education and Training (TVET) in Nigeria. Proceedings of the 2nd International Conference, The Federal Polytechnic, Ilaro,* 10th – 11th Nov., 689-694.

Alawadhi S. & Konsowa M. (2007). *Application of Absorbing Markov Analysis to student flow in an academic institution. Kuwait journal of Science and Engineering. 34*(2A), 77-89.

Amaghionyeodiwe L.A. & Osinubi T.S. (2006). *The Nigerian Educational System and Returns to Education. International Journal of Applied Econometrics and Quantitative Studies. 3*(1), 31-40.

Aparna B. & Sarat C.K. (2017). *A Stochastic Process Approach to Analyse Students' Performance in Higher Education Institutions. International Journal of Statistics and Systems. 12*(2), 323-342.

Brezavšček, A., Bach, M.P. & Baggia A. (2017). *Markov Analysis of Students' Performance and Academic Progress in Higher Education, Organizacija, 50*(2),83-95.

Bartholomew D. J. (1973). Stochastic Models for Social Processes. 2nd edn. London, New York, Sydney and Toronto: John Wiley and Sons.

Ezugwu V.O. & Ologun S. (2017). *Markov Chain: A Predictive Model for Manpower Planning. Journal of Applied Science and Environmental Management, 21*(3) ,557-565.

Faria A., Sorensen N., Heppen J., Bowdon J., Taylor S., Eisner R. & Foster S. (2017). Getting students on track for graduation: Impacts of the Early Warning Intervention and Monitoring System after one year. National Center for Education Evaluation and Regional Assistance.

Federal Republic of Nigeria (2013). National policy on education. Lagos; NERDC.

Hlavatý R. & Dömeová L. (2014). *Students' Progress throughout Examination Process as a Markov Chain. International Education Studies. 7*(12), 20-29.

Idiata D.J., Edoimioya, P.O. & Aigbomian, S.E. (2008). *The Problems of Technology Education in Nigeria: A Look into Polytechnic Education. Nigerian Academic Forum. 14*(1), 88-93.

Idoko, E.I. (2005). Repositioning Polytechnic Education in Nigeria. A Conference Paper. National Conference of the National Association for Research Development.

Kemeny, J.G. & Snell, J.L. (1967). Finite Markov Chains, With a New Appendix Generalisation of a Fundamental Matrix, Springer –Verlag New York.

Ledwith, M.C. (2019). Application of Absorbing Markov Chains to the Assessment of Education Attainment Rates within Air Force Materiel Command Civilian Personnel. Unpublished Thesis. Department of Operational Sciences Graduate, School of Engineering and Management, Air Force Institute of Technology Air University.

Mavruk C. & Ersin K. (2016). *Academic Progress of Students in Quantitative Courses at Nigde University Vocational School of Social Sciences: A Prediction Using Markov Model. Niğde Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, Temmuz; 9*(3), 267-276.

Murtala A.M., Jamilu Y.F. & Usman H.S. (2019). *Analysis Of Students' Academic Performance and Progression (Using Markov Chain Approach). International Journal of Engineering Applied Sciences and Technology, 4(*7), 187-193.

Musiga, L., Owino, J. & Weke P. (2011). *Modeling a hierarchical system with double absorbing States International Journal of Business and Public Management. 1*(1), 66-69.

National Board for Technical Education (NBTE) (2001). Digest of Statistics on Polytechnics in Nigeria (1998 - 2000). Kaduna NBTE.

National Board for Technical Education (2014). Guidelines and Procedures for The Establishment of Private Technical and Technological Institutions in Nigeria, Kaduna – Nigeria.

Odukoya J.A., Bowale E. & Okunlola S. (2018). *Formulation and Implementation of Educational Policies in Nigeria. African Educational Research Journal. 6*(1), 1-4.

Omotosho O.J. (2013). *Evaluation of Grading Systems of Some Tertiary Institutions in Nigeria. Information and Knowledge Management. 3*(2), 92-124.

Ovbiagele A.O. (2015). *Vocational Education for Socio-Economic and Technological Development of Nigeria, Global Journal of Interdisciplinary Social Sciences, 4* (4),15-18.

Pierre C. & Silver C. (2016). *Using a Markov Chain Model to Understand the Behavior of Student Retention. Proceedings of IMCIC – ICSIT, Atlanta USA*. 248-251

Shah C. & Burke G. (1999). *An undergraduate student flow model: Australian higher education. Higher Education 37*(1), 359-375.

Shahab B. & Adan E.V. (2020). *Improving Graduation Rate Estimates Using Regularly Updating Multi-Level Absorbing Markov Chains. Journal of Educational Sciences, 10*(377), 1-18.

Yakubu I.K., Shamsuddeen A.S., Ibrahim Z.M. & Ibrahim J. M. (2020*). Modeling of Student's Academic Performance and Progression in Indian Private Universities Using Markov Chain. International Journal of Research in Engineering & Science. 4*(6), 1-7.