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SOFT BCK/BCI-EXTENDED-POSITIVE IMPLICATIVE IDEAL OF SOFT BCK/BCI-ALGEBRAS

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Abstract: *This paper introduced the idea of soft BCK/BCI-ext-positive implicative ideals and BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebras and discussed some of their fundamental characteristics soft ideals and soft BCK/BCI-ext-positive implicative affairs of soft BCK/BCI-ext-algebras are also recognized. Also soft idealistic algebraic BCK/BCI-ext and implicative idealistic soft BCK/BCI positive Algebras with BCK/BCIs are connected. The intersection, union, "AND" and "OR' operation of Soft BCK/BCI-ext-positive implicative ideals and BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebras were initiated. BCK/BCI-ext-homomorphism were also demarcated.*

Keywords: *Implicative, positive implicative ideal, BCK/BCI-ext-positive implicative, BCK/BCI-ext-homomorphism.*

INTRODUCTION

The model of soft set is fundamental and is an significant mathematical apparatuses. In light of numerous inherent uncertainties, classical methods fall short in addressing the complexities of problems in engineering, economics, and the environment. The rough set theory and internal mathematics theory, both of which are employed, have their own set of challenges when it comes to dealing with uncertainties. One of the causes of these challenges is the insufficiency of the theories of parameterization tool, as (Molodtsov 1999) pointed out. Molodtsov devised the concept of soft sets as new mathematical tools to address these issues dealing with unpredictability. A parameterized general mathematical set is known as a soft set.

In the study of soft set theory, the objects initial description is only approximation. There is no need to bring up the concept of an exact soft set theory is particularly convenient and easy to apply in reality because there are no limits on the approximate description. Utilization of soft set theory is now catching on in a variety of areas and in real world issues.

The notion of a vague set was developed by (Xu *et al.* 2010) as an extension of soft sets and the concept of the intersection of two soft sets defined by (Maji *et al*. 2003). (Zou and Xiao 2008) provided data analyst methodologies for the soft set insufficient information. (Atagun and

Sizgin 2011) introduced the concept of normalistic soft group and normalistic soft group homomorphism and discussed some structures that are preserved under normalistic soft group homomorphism. (Sezgin and Atagun 2011) introduced soft sub-rings and soft ideals of ring. Soft subfield of a field and soft sub-module of a left R-module were also introduced.

However, they also investigated properties related to soft substructure of rings, field and module. (Ibrahim and Yusuf 2012) gave further information regarding development of soft set theory. (Neong, 2012) made and attempt to solve a decision making problem using imprecise soft set by considering a hypothetical case study.

(Park *et al* 2012) studied the equivalence soft set relations and obtained soft analogues of many results concerning ordinary equivalence relations and partitions. (Jun *et al.* 2009) introduced the notion of P-idealistic soft BCK/BCI -algebras. (Jun 2008) applied the concept of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft sub-algebras. (Jun *et al* 2013) introduced a new notion, called an internal and external cubic set and investigated several properties. (Cogman and Enginoglu 2011) introduced fuzzy parameterized soft sets and their related properties. They proposed a decision making method based on fuzzy parameterized soft set theory and provided an example which shows that the method can be successfully applied to the problems that contain uncertainty. (Feng 2008) studied the basic concept of soft set theory and compared soft sets to fuzzy and rough sets providing example to clarify their differences. (Chen *et al.* 2007) presented a new definition of soft set parameterized reduction and compared this definition to relate concept attributes reduction in a rough set theory.

1.0 SOME BASIC DEFINITION OF SOFT SET

In (Molodtsov 1999) the soft set is defined in the following way: Let $\cup$ be an initial universal set and E be a set of parameters. Let $\cup$ denotes the power set of $\cup$ and $A \subset \in$.

Definition 1.1

(Molodtsov 1999). A pair (F, A) is called a soft set over $\cup$, where F is a mapping given by:

$$F: A \to \delta(\cup)$$

In other words, a soft set over $\cup$ is a parameterized family of subsets of the universe $\cup$. for $a \in A$, $F(a)$, may be considered as the set of a – approximate elements of the soft set (F, A).

Definition 1.2

(Maji *et al.,* 2003). Let $(F, A) and (G, B)$ be two soft sets over a common universe $\cup$. The intersection of $(F, A) and (G, B)$ is defined to be the soft set (H, C) satisfying the following condition:

i. $C = A \cap B$
ii. $H(x) = F(x) or G(x) for all x \in C$

$$\therefore (F, A) \cap (G, B) = (H, C)$$

Definition 1.3

(Maji *et al.,* 2003). Let $(F, A) and (G, B)$ be two soft sets over a common universe $\cup$. The union of $(F, A) and (G, B)$) is defined to be the soft set (H, C) satisfying the following condition:

i. $C = A \cup B$
ii. $For all x \in C$

$$H(x) = \begin{cases} F(x) if x \in A|B \\ G(x) if x \in B|A \\ F(x) \cup G(x) if x \in A \cap B \end{cases}$$

$(F, A) \cup (G, B) = (H, C)$.

2.0 EXTENDED BCK/BCI-EXT- POSITIVE IMPLICATIVE IDEAL

The BCK/BCI-ext-algebras are considered to be a very important classes of logical algebras and these algebras was initiated by Imai and Iseki (1996) and were widely studied by various scientists.

Definition 2.1 (Malik and Touqeer, 2014)

An algebra $(Y,*,0)$ of type $(2,0)$ is called a BCK/BCI-algebra if it satisfies the following conditions:

i. $\left((d*e)*(d*f)\right)*(f*e)=0$
ii. $\left(d*(d*e)\right)*e=0$
iii. $d*d=0$
iv. $d*e=0$ and $e*d=0 \Longrightarrow d=$
v. $0*d=0$, for all $d,e,f\in Y$. If

EXAMPLE 2.2

Assume that $(Y;\leq)$ is a partially ordered set with the least element 0. Define an operation $*$ on Y by

$$d*e=\begin{cases}0 & if d\leq e\\ \\ d if d & \nleq e.\end{cases}$$

Then $(Y;*,0)$ is a BCI-algebra. It is obvious that axiom 2 and 3 is hold,

To verify 3 let $d,e,f\in Y$.If$d\leq e$,then

$$\left((d*e)*(d*f)\right)*(f*e)=\left(0*(d*f)\right)*(f*e)=0*(f*e)=0.$$

If $d\leq e$ and $d\ \nleq f$, then

$$\left((d*e)*(d*f)\right)*(f*e)=(d*d)*(f*e)=0*(f*e)=0.$$

If $d\ \nleq f$,and $d\leq e$ so using transitivity of the partial ordering $\leq$ that $f\nleq e$.

Also, since 0 is the least element of Y, by $d\nleq e$, we have $d\neq 0$, $d\nleq 0$. Therefore

$$\left((d*e)*(d*f)\right)*(f*e)=(d*0)*f=d*f=0.$$

Definition 2.3

An algebra $(X,*,/,0)$ of type $(3,0)$ is called an extended BCK/BCI-ext-algebra denoted by BCK/BCI-ext-algebra if it satisfies all the conditions of BCK/BCI-ext-algebras and

vi. $d*\frac{(d*e)}{d*e}*e=0, \forall\, d,e\in Y$

In an extended BCK/BCI-ext-positive implicative ideal we can introduce a partial ordering " $\leq$ " by saying $d\leq e\Leftrightarrow d/e = e$ and $d\leq e\Leftrightarrow d*e= 0$

In an extended BCK/BCI-ext-positive implicative ideal if the following hold:

vii. $(d * e) * f = (d * f) * e$
viii. $(e * e)/f = (e/f) * e$
ix. $e/0 = 0$
x. $e * 0 = e$
xi. $d \leq e \Rightarrow d * f \leq e * f$
xii. $d \leq e \Rightarrow e/f \leq e/f$. for all $d, e, f \in Y$

Definition 2.4

A non-empty subset M of a BCK/BCI-ext-algebra X is called a sub-algebra of X if $x * y \in M$ and ${}^{e}/_{d} \in M$ for all$d, e \in M$.

Definition 2.5

A non-empty subset I of a BCK/BCI-ext-algebra X is called an ideal of X if for any $x \in X$

(I_1) $0 \in I$

(I_2) $d * e \in I$

(I_3) ${}^{d}/_{e} \in I$

Definition 2.6

(Iseki 1975), (Liu and Zhang 1994)

A subset A of BCK/BCI-algebra X is called positive implicative ideal of X ifsatisfies

i. $0 \in A$
ii. $(d * e) * f \in A$ and $e * f \in A$ imply $d * f \in A$ for any $d, e, f \in A$.

Definition 2.7

A non-empty subset I of a BCK/BCI-ext-algebra Y is called a BCK/BCI-ext-positive implicative ideal of Y if it satisfies the following condition.

PI1: $0 \in I$

PI2: ${}^{d}/_{e} \in I$

PI3: $\big((d * f) * f\big) * (e * c) \in I$ and $Y \in I$

$\Rightarrow x * z \in I$ for all $x,\ z \in X$.

Theorem 2.8

A BCK/BCI-ext-algebra Y is positive implicative ideal if and only if $(e * f) * f \in A$ imply $d * f \in A$ for all $d, e, f \in Y$.

Proof.

First of all to proof we need to show: If X is a positive implicative ideal and $(e * f) * f \in A$ then $e * f \in A$, then Y is a positive implicative ideal.

First, let's prove the forward implication (1):

Assume y is a positive implicative ideal and $(e * f) * f \in A$. We need to show $e * f \in A$.

Since Y is a positive implicative ideal, for any elements $d, e, f \in Y$. Therefore $d \leq e \leq f$ then $d * f \leq e * f$ if $d \leq e \leq f$.

Applying this property to our case, we have:

$(e * f) * f \in A$ (Given)

$e * f \leq (e * f) * f$ (By definition of partial ordering

$e * e \leq (e * f) * f$ (By transitivity)

Therefore, $e * f \in A$. Hence A is positive implicative ideal

Proposition 2.9

Suppose that $(Y{:}\backslash, * ,0)$ is a BCK/BCI-ext-algebra. Defined to be a binary relation $\geq and \leq$ on Y by which $e \geq d$ and $d \leq e$ if and only if $e * d = 0$ and $e * d = 0$ for any $d, e \in Y$. Then $(Y{:} \geq, \leq)$ is a partially systematic set with 0 as a minimal component in the meaning that $e \geq 0 \; and \; 0 \leq e \therefore e = 0$ for any $e \in Y$.

Proof.

For any $d, e, f \in Y$, we have $e * e = 0$ then $e \geq e$ which is reflexivity. If $d \geq e$ and $d \geq e$ or $e \leq dandd \leq e$, then $d * e = 0 \; ande * d = 0 \; soe = d$. which axiom (3) is and symmetric.

Also if $e \geq dandd \geq f$ or $e \leq dandd \leq f$, then $d * e = 0$ and $d * f = 0$

axiom (1) and (2) $e * f = \big((e * f) * 0\big) * 0 = \big((e * f) * (e * d)\big) * (d * f) = 0$, hence $e \leq f$ and transitivity is Hold.

Finally, if $e \geq 0, or \; 0 \leq e$, $\; 0 * eore * 0 = e$. Hence $e = 0 \; and \; 0$ is the least element of Y.

Theorem 2.10

An algebra $(X{:} \; *, \backslash, 1)$ of type (3,1) is known as BCK/BCI-ext-algebra, if an only if the following condition is satisfies

i. $\big((d * e) * (d * f)\big) * (f * e) = 0$
ii. $d * 0 = d$
iii. $d * \frac{(d*e)}{d*e} * e = 0$

Define binary relation $*$ on Y. And denote the $d * e = de^{-1}$ For any $d, e, f \in X$.

From (i) $\big((d * e) * (d * f)\big) * (f * e) = 0$

$$\big((de^{-1}) * (df^{-1})^{-1}\big) * (fe^{-1})^{-1} = 1$$

$de^{-1}.d^{-1}f.fe^{-1} = 1$ (1) is holding

Also since $d * 1 = d1^{-1} = d$ (2) is also holding

Finally $d * \frac{(de^{-1})}{de^{-1}} * e$ =e

$\therefore d * e =$ e and let 1 = e

de^{-1} =e therefore $d = e$ (3) is hold.

3.0 SOFT BCK/BCI-EXT-POSITIVE IMPLICATIVE IDEAL

Let$Y$ be an extended BCK/BCI-ext-algebra and A be a non-vacuous set and R refer to an indiscriminate relation among an element of A and element of Y, that is, R is $R \subseteq A \times Y$ a set of valued function $F: A \to P(x)$ can be defined as $F(a) = \{b \in X/aRb\}$ for all $e \in A$ the pair (P, N) is then a soft set on Y.

Definition 3.1

A BCK/BCI-ext-algebra Y is called positive implicative if the following condition is satisfied

i. 0$\in A$

ii. $(d * e) * e = (d * f) * (e * e)$ for any $d, e, f \in Y$.

Proposition 3.2

If and only if, a BCK/BCI-ext-algebra Y is positive implicative.

$(e * f) * f = e * f$ for any $e, f \in Y$.

Proof

Assume that Y is positive implicative ideal and $d, e, f \in X$.

$(e * f) * f = (e * f)(f * f)$, since $f * f = 0$ and $0 \in A$

Hence $e * f = (e * f) * f$

Definition 3.3

Jun and Park (2008). Let S be a sub-algebra of X. A subset I of X is called an ideal of X related to S (briefly, S – ideal of X), denoted by $I \tilde{\trianglelefteq} S$, if it satisfies.

i. $0 \in I$

Definition 3.4

Let M be a sub-algebra of X. A subset I of X is called a BCK/BCI-ext-positive implicative ideal of X related to M, denoted by $I \lhd_{bck-bci-ext-p} M$, if it satisfies.

i. $0 \in I$

ii. $d/e \in I$ and $d \in I \Rightarrow e \in I$

iii. $\big((d * f) * f\big) * (e * f) \in I$ and $y \in I \Rightarrow d * f \in I$ for all $d, f \in M$

EXAMPLE 3.5

Let $Y = \{0, q_1, q_2, q_3, q_4, q_5\}$ be a BCK/BCI-ext-algebra with the following Cayley tables:

Cayley Table 1:

*	0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
0	0	0	0	0	0	0	0	q_7
q_1	q_1	0	0	0	0	0	0	q_7
q_2	q_2	q_2	0	0	0	0	0	q_7
q_3	q_3	q_3	q_3	0	0	0	0	q_7
q_4	q_4	q_4	q_4	q_4	0	0	0	q_7
q_5	q_5	q_5	q_5	q_5	q_5	0	0	q_7
q_6	q_6	q_6	q_6	q_6	q_6	q_6	0	q_7
q_7	q_7	q_7	q_7	q_7	q_7	q_7	q_7	0

Cayley Table 2:

/	0	q_1	q_2	q_3	q_4	q_5	q_6	a_7
0	0	q_1	q_2	q_3	q_4	q_5	q_6	0
q_1	0	a_1	q_2	q_1	q_1	q_1	q_1	0
q_2	0	0	q_2	q_2	q_2	q_2	q_2	0
q_3	0	0	0	q_3	q_3	q_3	q_3	0
q_4	0	0	0	0	q_4	q_4	q_4	0
q_5	0	0	0	0	0	q_5	q_5	0
q_6	0	0	0	0	0	0	q_6	0
q_7	0	0	0	0	0	0	0	q_7

Then $M = \{0, q_1, q_2,\}$is a sub-algebra of Y and $l = \{q_1, q_2, q, q_4\}$ is an M-BCK/BCI-ext-ideal of Y.

Definition 3.6

Let $(Q, S,)$ be a soft set over Y. then (Q, S) is called a soft BCK/BCI-ext-algebra over Y if $F(d)$ and $F(d/e)$ are sub-algebra of Y for all $d, e \in A$.

Definition 7: Let (Q, S) be a soft BCK/BCI-ext-algebra over Y. A soft set (G, I) over Y is called a soft BCK/BCI-ext-positive implicative ideal of (Q, S) denoted by $(G, I) \lhd_{bck/bci-ext-pi} (Q, S)$, if it satisfies.

i. $I \subset A$
ii. $G\left({}^{d}/_{e}\right) \lhd F({}^{d}/_{e})$ for all $d, e \in I$
iii. $G(d) \lhd F(d)$ for all $d \in I$

Definition 3.7

Let (Q, S) be a soft BCK/BCI-ext-algebra over Y. A soft set (G, I) over Y is known as soft ideal of (Q, S) denoted by $(G, I) \lhd_{ext} (Q, S)$ if it satisfies.

i. $I \subset A$
ii. $G\left({}^{d}/_{e}\right) \lhd_{ext.} F\left({}^{e}/_{d}\right)$ for all $d, e, \in I$
iii. $G(d) \lhd_{ext.} F(d)$ for all $d \in I$

Example 3.8

Let$Y = \{0, q_1, q_2, q_3\}$ be a soft BCK/BCI-ext-algebra with the following Cayley table.

Table 3:

*	0	q_1	q_2	q_3
0	0	0	0	q_3
q_1	q_1	0	0	q_3
q_2	q_2	q_2	0	q_3
q_3	q_3	q_3	q_3	0

Table 4:

/	0	q_1	q_2	q_3
0	0	q_1	q_2	0
q_1	0	q_1	q_1	0
q_2	0	0	q_2	0
q_3	0	0	0	q_3

Let (F, A) be a soft set over X, where $A = X$ and $F: A \to \beta(x)$ is a set valued function defined by:

$$F(x) = \{0\} \cup \left\{y \in X | {}^{x}/_{y} \in \{0, q_1, q_3\}\right\}$$

$\therefore F(0) = Y,\ F(q_1) = \{0, q_1, q_3\}, F(b) = \{0, q_3\}, F(q_3) = \{0, q_3, \}.$ for all $x \in A$.

$$\therefore F(0) = Y, F(q_1) = \{0, q_1, q_3\}, F(q_2) = F(q_3) = \{0, q_3\}$$

Let (G, I) be a soft set over X, where $I = \{b, c\} \subset A$ and $P(x)$ be a set valued function defined by $G(x) = \left\{y \in X | {}^{x}/_{y} = 0\right\}$

$$\therefore G(q_2) = \{0, q_1, q_3\}$$

$$G(q_3) = Y$$

$G(x) = \left\{y \in X | {}^{x}/_{y} = 0\right\}$ for all $x \in I$.

$G(q_2) = \{0, q_1, q_3\} \lhd (0, q_1, q_3 0, q_1, q_3) = F(q_1), G(q_3) = x \lhd (0, q_3) = F(c)$. Hence (G, I) is a soft BCK/BCI-ext-positive implicative ideal of (F, A)

Theorem 3.9

Let (P, N) be a soft BCK/BCI-ext-algebra over (Y). Any soft set $(H, I)(G, B)$ and (K, I) over X. Where $I \cap W \cap J \neq 0$.

We have, (H, I) is a soft BCK/BCI-ext-positive implicative ideal of (P, N),

(L, W) is a soft BCK/BCI-ext-positive implicative ideal of (P, N).

(K, J) is a soft BCK/BCI-ext-positive implicative ideal of (P, N).

Proof

From Definition

Let$(U, Z) = (H, I,) \cap (L, W,) \cap (K, J,)$

Where $Z = I \cap W \cap J$

$\therefore H(e) = L(e) or K(e)$ for all$e \in I$

Obviously $Z \subset A$.

$D: I \to \delta(x)$ mapping. Hence (U, Z) is a soft set over Y.

Since $(H, I,) \lhd_{bck|bci-ext-P} \ (P, N)$

$(L, W) \lhd_{bck|bci-ext-Pi} (P, N)$

$(K, J) \lhd_{bck|bci-ext-Pi} (P, N)$

it follows that

$H(e) = A(e)$ is a soft BCK/BCI-ext-positive implicative ideal of $P(e)$ or

$L(e) = W(e)$ is a soft BCK/BCI-ext-positive implicative ideal of $P(e)$ or

$K(e) = J(e)$ is a soft BCK/BCI-ext-positive implicative ideal of $P(r)$or

Hence $(H, I,) \cap (L, W) \cap (K, J) = (U, Z)$ is a soft BCK/BCI-ext-positive implicative ideal of(P, N).

Corollary 3.10

Suppose (P, N) is a soft BCK/BCI-ext-algebra over X. Any soft sets (G, I_1) ,(G, I_2),$and (G, I_3)$ over X. We have

$$(G_1, I_1) \lhd_{bck|bck-ext-Pi} (P, N)$$

$$(G_2, I_2) \lhd_{bci|bci-ext-Pi} (P, N)$$

$$(G_3, I_3) \lhd_{bck|bci-ext-P} \ (P, N)$$

$\Rightarrow (G_1, I_1) \cap (G_2, I_2) \cap (G_3, I_3) \lhd_{bck|bci-ext-Pi} (P, N)$.

Theorem 3.11

Suppose that (P, N) be a soft BCK/BCI-ext-algebra. Let $(Q, V) and (R, W)$ be a soft set over X. such that $A and B$ have element in common.

Let $V and W \neq 0$

$$\therefore (Q, V) \lhd_{bck|bci-ext-P} \ (P, N)$$

$$(R, W) \lhd_{bck|bci-ext-P} \ (P, N)$$

$$\Rightarrow (Q, V) \cap (R, W) \lhd_{bck/bci-ext-Pi} (P, N)$$

Proof

Let $(Q, V) \lhd_{bck/bci-ext-p} \ (P, N)$ and

$$(R, W) \lhd_{bck/bci-ext-Pi} (P, N)$$

From above

$$(Q, V) \cap (R, W) = (K, X)$$

$$X = V \cap W$$

$$K(x) = Q(x) or R(x)$$

∴ In this case $(Q, V) \cap (R, W) = K(x) \lhd_{bck/bci-ext-Pi} \quad (P, N)$

Example 3.12

Let$Y = \{0, q_1, q_2, q_3, q_4\}$ be a BCK-algebra and hence a BCI-algebra with the following Cayley tabl

Table 5:

*	0	q_1	q_2	q_3	q_4	q_5
0	0	0	0	0	q_4	q_5
q_1	q_1	0	q_4	0	0	q_4
q_2	q_2	q_3	0	0	q_2	q_4
q_3	q_3	q_3	q_3	0	0	0
q_4	q_4	q_4	q_4	q_4	0	0
q_5	q_5	q_5	q_5	q_5	q_2	0

Table 6

*	0	q_1	q_2	q_3	q_4	q_5
0	0	q_1	0	0	q_4	0
q_1	0	q_1	q_2	q_3	0	0
q_2	q_2	q_2	0	0	q_4	0
q_3	0	q_3	q_3	0	0	q_5
q_4	0	q_4	q_4	q_4	0	0
q_5	q_5	q_5	0	0	q_5	0

Let (P, N) be a soft set over , where $A \subset X$ and such that $A = \{0, q_1, q_2, q_3, q_4\}$ and $P: N \rightarrow \delta(x)$ is a set value function defined by:

$$F(x) = \left\{y \in X | {}^{x}/_{y} \in \{0, q_2, q_3\}\right\}$$

$$F(0) = \{0, q_1, q_2, q_3, q_4\}$$

$$F(q_1) = \{0, q_2, q_3\}$$

$$F(q_2) = \{0, q_2, q_3, q_4\}$$

$F(q_3) = \{0, q_1, q_2, q_3\}$ for all $x \in A$.

Then, $F(q_3) = \{0, q_1, q_2, q_3, q_4\}, F(q_1) = \{0, q_2, q_3\}, F(q_2) = \{0, q_2, q_3, q_4\}$ and $F(q_3) = \{0, q_1, q_2, q_3\}$ which are sub-algebras of Y. Therefore, (P, N) is a soft BCK/BCI-ext- algebra over Y.

Let (U, Z) be a soft set over X, where $I = \{q_2, q_3, q_4\} \subset A$ and $G: I \rightarrow \delta(x)$ be a set value function defined by:

$$G(x) = \left\{y \in X | {}^{x}/_{y} \in \{0, q_3\}\right\}$$

$$G(q_1) = \{0, q_3\}$$

$$G(q_2) = \{q_2, q_3\}$$

$$G(q_3) = \{0, q_1, q_2, q_3\}$$

$\therefore$ for all $x \in I$ then $G(q_1) = \{0, q_3\} \lhd_{bck/bci-ext-Pi}$ $(0, q_2, q_3) = F(q_2), G(q_2) = \{q_2, q_3\} \lhd_{bck/bci-ext-Pi} (q_2, q_3, q_4) = F(q_2), G(q_3) =$

$\{0, q_1, q_2, q_3\} \lhd_{bck/bci-ext-Pi} (0, q_1, q_2, q_3) = F(q_3)$. However, (G, I) is a soft BCK/BCI- ext-positive implicative ideal of (P, N).

BCK/BCI- ext-positive implicative idealistic soft BCK/ BCI-ext- algebra.

Definition 3.12

(Jun and Park 2008). Let (P, N) be a soft set over Y. Then (P, N) is called an idealistic soft BCK/BCI -ext-algebra over X if $F(x)$ is an ideal of Y for all $x \in A$.

Definition 3.13

Suppose that (P, N) be a soft set over Y, if $P(x)$ is a BCK/BCI-ext-positive implicative ideal of X, then $(P, N,)$ is called BCK/BCI-ext-positive idealistic soft BCK/BCI-ext-algebra over Y.

Theorem 3.14

Suppose that two finite set be BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebra over Y. Then their differences is also a BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebra over Y.

Proof:

Let the two finite set be disjoint set (no element in common).

$\therefore U\backslash W = U$ and $L\backslash W = W$ from their relation knows that

$U - W = \phi$ if $U \cap P = \phi$

$W - U = \phi$ if $W \cap P = \phi$

Also $U - W = \phi$ if $U \subset W$

The set difference of $UandW$ is the set $U\backslash W = \{x: x \in A \wedge x \notin B\}$ for all $x \in A$.

For symmetric difference of $UandW$.

$$U * W = (U\backslash W) \cup (W\backslash U) = (x \in A \wedge x \notin B) \vee (x \in B \wedge x \in A)$$

$\therefore U\backslash W = U$ and $W\backslash U = W$

$\Rightarrow UandW$ are two BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebra.

Theorem 3.15

For any two BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebra over X, the union is also BCK/BCI-ext- positive implicative idealistic soft BCK/BCI-ext-algebra on Y. Let the two soft set be $(H, I,) \cup (W, V) = (U, Z)$ provided $I \cup V \neq 0$.

Proof:

$$\therefore U(e) = H(e) \vee W(e) \forall\ e \in Z$$

(U, Z) is a soft set over X, since both set are positive implicative idealistic soft BCK/BCI-ext-algebra on Y, $H(e) and W(e)$ is also positive implicative ideal of $X\ \forall\ e \subset Z$. Hence $(U, Z) = (H, I) \cup (W, V)$ is also BCK/BCI-ext-positive implicative idealistic soft BCK/BCI-ext-algebra.

4.0 BCK/BCI-EXT-HOMOMORPHISM

Definition 4.1

Suppose that $(Y; *, 0)$ and $Y; \cdot, 0)$ be two BCK/BCI-ext-algebra. A mapping $U: V \to W$ is called BCK/BCI – ext-homomorphism if:

$U(v * w) = F(v) * F(w)$ for all $v, w \in G$

Example 4.2

Using an algebra $(Y, *, 0)$ let X is the set $\{0,1,2\}$ define an operation who Cayley table are as follows shows that X is BCK/BCI-ext-homomorphism

*	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

Assume that

$$\big((1 * 2) * (1 * 0)\big) * (0 * 2)$$

$$= (0 * 1) * 2$$

$$= 0 * 2 = 0$$

Hence is a BCK/BCI-ext-homomorphism.

Theorem 4.3

Let$F: X \to Y$ be a monomorphic of BCK/BCI-ext-algebras. If $g_1 g_2$ is a BCK/BCI-ext-positive implicative ideal of Xshows that $g_1 g_2: z \to x$ is a BCK/BCI-ext-positive implicative ideal of X.

Proof:

For any BCK/BCI-ext-Homomorphism

$g_1 g_2: z \to x \therefore F g_1 = F g_2$ by positivie implication. $F\big(g_1(z)\big) = F g_1(z) = F g_2(z) = F(g_2(z)) \forall\ z \in Z$.

Meanwhile F is one to one we obtain $g_1(z) = g_1(z)$. Hence $g_1 = g_2$ is BCK/BCI-ext-positive implicative ideal.

Corollary 4.4

Let$F; G \rightarrow H$ it is a BCK/BCI-ext-homomorphism, then F is injective.

Proof

Assume that f is a function an has a domain is H.

For all $x_1, x_2 \in H$, therefore $f(x_1) = f(x_2)$

Implies that $x_1 = x_2$. $f$is injective.

Hence F is a BCK/BCI-ext-homomorphism.

CONLUSION.

The attribute of soft set theory was designed by man called Molodtsov (1999). He described the concept as a new mathematical tools for dealing with uncertainties which is free from any difficulties. In this paper we introduced the idea of soft BCK/BCI extended positive implicative ideals and the idea of soft BCK/BCI extended positive implicative idealistic soft BCK/BCI extended algebras. We also talked about idea of the Intersection, Union, "AND" and "OR" operation of soft BCK/BCI extended positive implicative ideals and the idea of soft BCK/BCI extended positive implicative idealistic soft BCK/BCI extended algebras.

Finally, we established the notion soft BCK/BCI extended Homomorphism of soft BCK/BCI extended algebras.

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