Network for Research and Development in Africa
International Journal of Pure and Applied Science Research
ISSN: 2384-5918, Volume 11, Issue 13
PP 57-68 (July, 2024)
DOI: 45727711-11135
arcnjournals@gmail.com
https://arcnjournals.org

Modeling the Nigerian Stock Exchange Market Using a Modified Garch Model

Mustapha, Hassan
Department of Statistics, Federal Polytechnic Monguno, Borno state

***Abstract**: Stock exchange exhibit changes in variance over time because of the volatility. Volatility is a measure of dispersion for a given security or market index. GARCH model is most effective model in modeling and forecasting a volatility in financial series return but it's widely known that GARCH model is normally distributed and financial series return generally exhibit a non-normal characteristic and it cannot capture influence in each variance. In this research we modified GARCH(p,q) model with log-normal distribution and recommend procedure for selecting an order of model with the distribution is not normal. The result shows that GARCH model with log-normal outperformed the GARCH(p,q) with normal and generalized error distribution base on the MSE and AMSE because they have the minimum values and minimum information criterion.*

***Keywords:** GRACH, ARCH, Log-Normal, volatility, stationarity.*

- **INTRODUCTION**

Stock market exhibit changes in variance over time. in such circumstances, that the assumptions of constant variance(homoscedasticity) is inappropriate. The variability in the financial data could very well be due to the volatility of the financial market. More importantly, the extended financial market as well as globalization due to the markets is known to be sensitive to factors such as rum ours, political upheavals and changes in the government monetary and fiscal policies (Usman *et. al.,* 2018). (Bollerslev, 1986) Introduced the Autoregressive Conditional Heteroscedastic (ARCH) model process to cope with the changing variance and also extended the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model which has a more flexible lag structure because the error variance can be modeled by an Autoregressive Moving Average (ARMA) type process. Such a model can be effective in removing the excess kurtosis. There have been a great number of empirical applications of modeling the conditional variance (volatility) of financial time series by employing different specifications of these models and their many extensions. Volatility is defined as a measure of dispersion of returns for a given security or market index in time series, financial data generally exhibit a non-normal characteristic but typically leptokurtic and exhibits heavily tall behavior. To solve these problems, several solutions were employed for the distribution. Some of these literatures are (Liu, Lee, and Lee,2009) he examined the performance of volatility forecasting on daily prices of Shanghai and Shenzhen composite stock indices using (GARCH (1,1) models with N and SGED). Their results confirm that the GARCH model with SGED is superior to the GARCH model with normal distribution. Also (Chaiwat, 2013) develop a GARCH model with six different error distribution and compare with the normal distribution. The results indicate that GARCH(p,q) models with non-normal distributions outperform GARCH(p,q) models with a normal distribution. In the same vain (Lingbing &Yanlin, 2017) develop a GARCH model with tempered stable distribution and compare the model with normal, student-t and GED distributions using S&P 500 daily return for Australia. They argue that the tempered stable distribution outperforms the normal, student-t and GED distributions also, (Khushnoor, 2019) used three set of error distribution for comparing the predictive ability of GARCH (1,1) model i.e. normal, student-t and GED distributions. He found that GARCH (1,1) with generalized error distribution outperform for capturing the volatility of Flying cement industry. Amiratul et al (2020), developed a GARCH model with Novel Fuzzy Linear Regression Sliding Window (FLR-FSWGARCH) model and compare the model with normal distribution. The result shows that GARCH (1,1) with fuzzy Linear Regression Sliding Window (FLR-FSWGARCH) model outer perform best.

- **METHODOLOGY**

Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model

Generalized GARCH model was proposed independently by Bollerslev (1986) and Taylor (1986) as it adds the lagged conditional variance (σ^2_{t-j}) to the ARCH model as a new in the GARCH model. The Generalized ARCH (GARCH) model also reduced the number of estimated parameters. And it's given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2 \quad - \quad 1$$

Where σ_t^2 will be replaced by h_t

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \quad - \quad 2$$

$$h_t = \alpha_1 \varepsilon_{t-1}^2 + \ldots\ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_j h_{t-1} + \ldots\ldots + \beta_p h_{t-p} \quad 3$$

Where h_t is the conditional variance and ε_{t-i}^2 is the past squared residual return also α_0>0 ,α_i>= 0, β_j>= 0 the above is GARCH(p.q) model.

- **GARCH model with non-normal error distribution**

The incompetence of the traditional GARCH model to capture volatility for some stylized fact for return series is widely known. The most outstanding disadvantage for the traditional GARCH with normal error distribution is that, those distributions areunable to capture the influence of each variance in the observation. And alsofails to capture stylized properties of underlying time series. Various non-normal error distributions have been suggested to solve these problems. The modification of GARCH model with alternative non-normal error distribution. many studies have been conducted previously to model and forecast volatility of the return series. Some previous studies are presented below; Also (Chaiwat, Kosapatta, rapim, 2013) develop a GARCH model with five different error distribution and compare with the normal distribution. The empirical results indicate that GARCH(p,q) models with non-normal distributions outperform GARCH(p,q) models with a normal distribution based on the three emerging indices from Thailand, Malaysia and Singapore. In same vain, also (Lingbing Feng &Yanlin Shi, 2017) develop a GARCH model with tempered stable distribution and compare the model with normal, student-t and GED distributions using S&P 500 daily return for Australia. They argue that the tempered stable distribution outperforms the normal, student-t and GED distributionsAlso (Khushnoor Khan, 2019) used three set of error distribution for comparing the predictive ability of GARCH (1,1) model i.e.normal, student-t and GED distributions. He found that GARCH (1,1) with generalized error distribution outperform for capturing the volatility of Flying cement industry. Researchers attempted to incorporate heavy tail distributions into GARCH models by adopting a variety of non-normal error distributions. The comparisons of competing GARCH models with complicated error distributions on the performance of volatility forecasting have been examined but this issue remains interesting. In this thesis, we will use lognormal distribution for the GARCH error term distribution and compare with the gaussian distribution and generalized error distribution. This thesis will not only be focused on the GARCH (1,1) model but also investigates whether it is more appropriate to fit the returns series of emerging stock markets into a higher order GARCH model in order to forecast the volatility of financial time series.

- **Simulation on Order Determination**

We can use simulation in order to choose the order of the GARCH (p,q) model under non-normality assumption. The set of observation for lognormal has a size of 2,000. The simulation started from GARCH of order (1,1), (1,2), (2,1), and (2,2) respectively. These four GARCH models are considered as the true GARCH models with error distributions mentioned above.
Where
$\alpha_1 = 0.3$
$\alpha_2 = 0.01$
$\beta_1 = 0.5$
$\beta_2 = 0.02$

The four sets of parameters in the true GARCH(p,q) models from Equation (3.5) are initially set as follows:

GARCH (1,1) model
$h_t = 0.01 + 0.3\varepsilon_{t-1}^2 + 0.5h_{t-1}$

GARCH (1,2)
$h_t = 0.01 + 0.3\varepsilon_{t-1}^2 + 0.5h_{t-1} + 0.02h_{t-2}$

GARCH (2,1)
$h_t = 0.01 + 0.3\varepsilon_{t-1}^2 + 0.01\varepsilon_{t-2}^2 + 0.5h_{t-1}$

GARCH (2,2)
$h_t = 0.01 + 0.3\varepsilon_{t-1}^2 + 0.01\varepsilon_{t-2}^2 + 0.5h_{t-1} + 0.02h_{t-2}$

The observations of each true GARCH(p,q) model are used to build up a model that assumed the normal error distribution. Coefficients in all models must be significant. Then, the AIC criterion is used to identify whether the order of GARCH model continue to correspond to the true models. If the order of each fitting GARCH model corresponds to the true GARCH model, it indicates that the order determination in each GARCH model with non-normal error distribution is valid.

- **RESULT AND DISCUSSION**

Table 1: ADF unit root test with respect to sample sizes for normally distributed data.

Sample size	Test values	Lag order	p-value	Hypothesis	Decision	Remark
200	-5.737	5	0.01	Unit root	Reject (H_0)	Stationary
400	-7.7001	7	0.01	Unit root	Reject (H_0)	Stationary
600	-7.6697	8	0.01	Unit root	Reject (H_0)	Stationary
800	-8.2617	9	0.01	Unit root	Reject (H_0)	Stationary
1000	-10.439	9	0.01	Unit root	Reject (H_0)	Stationary

Source: Authors' computation aided by R package v 4.1.1

Table 2: AIC and BIC values for GARCH (p,q) model when the error distribution is normal

	AIC				BIC			
ample size	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)
200	**2.724613**	**2.734565**	**2.733344**	**2.743344**	**2.790579**	**2.817023**	**2.815802**	**2.842294**
400	2.934338	2.939328	2.939328	2.944328	2.974252	2.989222	2.989222	3.004200
600	2.958330	2.961664	2.961664	2.964997	2.987643	2.998305	2.998305	3.008967
800	2.954825	2.957097	2.957097	2.959597	2.978248	2.986375	2.986375	2.994731
1000	2.821464	2.823464	2.823464	2.825386	2.841095	2.848003	2.848003	2.854832

Source: Authors' computation aided by R package v 4.1.1

Table 3: ADF test for unit root test with respect to sample sizes for GED data.

Sample size	Test values	Lag order	p-value	Hypothesis	Decision	Remark
200	-5.6255	5	0.01	Unit root	Reject (H_0)	Stationary
400	-7.2078	7	0.01	Unit root	Reject (H_0)	Stationary
600	-7.5726	8	0.01	Unit root	Reject (H_0)	Stationary
800	-8.8061	9	0.01	Unit root	Reject (H_0)	Stationary
1000	-10.144	9	0.01	Unit root	Reject (H_0)	Stationary

Source: Authors' computation aided by R package v 4.1.1

Table 4 AIC and BIC values for GARCH (p,q) model when the error is non-normally distributed (GED)

	AIC				**BIC**			
Sample size	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)
200	**2.181482**	2.691365	**2.691412**	2.694413	**2.447449**	2.773823	**2.773870**	2.793362
400	2.793293	2.798090	2.797491	2.802491	2.833207	2.847983	2.847384	2.862363
600	2.768506	2.771833	2.770612	2.771846	2.797819	2.80847	2.807253	2.815815
800	2.865924	**2.265664**	2.868245	2.868164	2.88934	**2.594943**	2.897704	2.903299
1000	2.746394	2.748195	2.748394	**2.680195**	2.766025	2.772734	2.772933	**2.779642**

Source: Authors' computation aided by R package v 4.1.1

Table 5: shows the estimated parameters and diagnostic of GARCG (1,1)-GED model.

Parameters	Generalized error distribution	p-values
Ω	1.439e^-01	0.0374*
α_1	1.000e^-04	0.0472**
β_1	1.000e^+00	<2e^-16***
ARCH(1)- LM test	5.893	0.0092
Q^2 (15)	27.6709	0.0237

Source: Authors' computation aided by R package v 4.1.1

Note: (*), (**) and (***) denote significance at 1%, 5% and 10% respectively.

Table 7:AIC and BIC values for GARCH (p,q) model when the error is non-normally distributed (log-normal distribution)

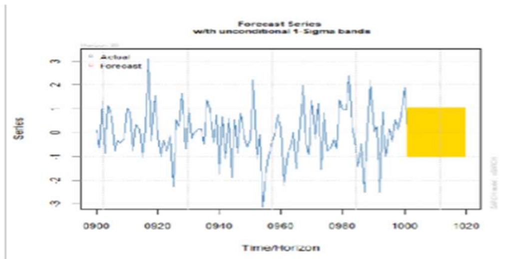
	AIC				**BIC**			
Sample size	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)	GARCH (2,2)
200	2.975542	2.489208	2.958529	2.460828	2.981507	2.921666	2.90837	2.659777
400	2.864287	2.362346	2.697540	2.374754	2.41801	2.889840	2.93948	2.534626
600	2.064887	2.069946	2.069754	2.074754	2.104801	2.119840	2.119648	2.134626
800	1.959303	**1.061778**	2.861172	**2.359597**	2.882726	**1.201057**	2.890450	**2.494731**
1000	**1.674126**	1.576100	1.875448	1.877448	**1.793757**	1.250639	1.887987	1.906895

Table 8: shows the estimated parameters and diagnostic od GARCG (1,2)-log-normal model

Parameters	Generalized error distribution	p-values
Ω	1.690933	<2e-16***
α_1	0.420181	0.0411*
β_1	0.004158	<2e^-16***
β_2	3.6338	0.0002**
ARCH(1)- LM test	5.893	0.09673
Q^2 (15)	27.6709	0.0237

Source: Authors' computation aided by R package v 4.1.1

Note: (*), (**) and (***) denote significance at 1%, 5% and 10% respectively



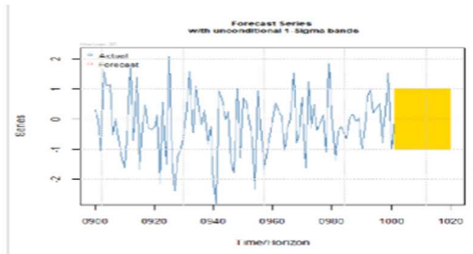


Fig 8: GARCH with normal Distribution forecast   *Fig 9: GARCH with GED forecast*

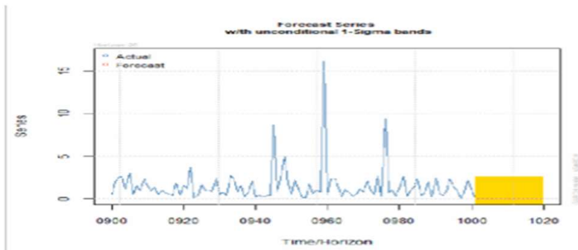


Table 10: Stationarity Test for The Data

Difference	Test values	Lag order	p-value	Hypothesis	Decision	Remark
0	-2.8908	3	0.2272	Unit root	Reject (H_1)	Not Stationary
1st	-2.7739	3	0.2732	Unit root	Reject (H_1)	Not Stationary
2nd	-3.6421	3	0.0447	Unit root	Reject (H_0)	Stationary

Source: Authors' computation aided by R package v 4.1.1

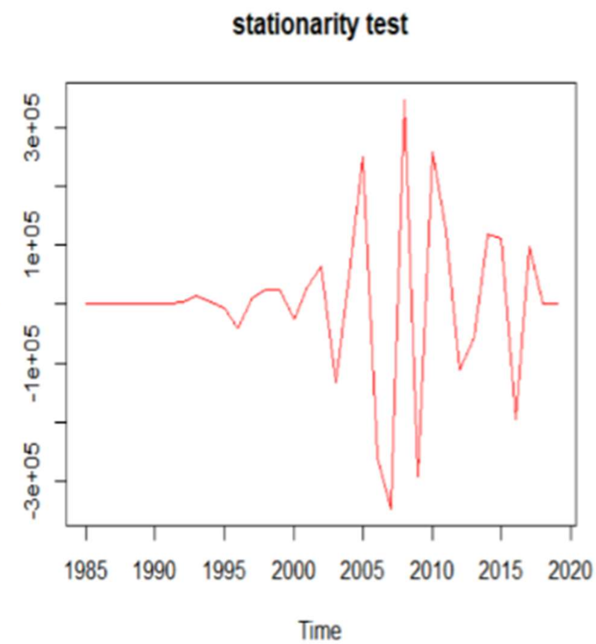


Table11: estimated parameters for GARCH(1,1) model with log-normal distribution

Estimated parameters			
Parameters	**LN**	**Std, Error**	**p-value**
Ω	0.0146	0.0828	0.0001**
α_1	0.0321	0.1578	0.0082*
β_1	0.0355	0.0808	0.0007*

Note: (*), (**) and (***) denote significance at 1%, 5% and 10% respectively.

Table 12 estimated parameters for GARCH(1,2) model with log-normal distribution

Estimated parameters			
Parameters	**LN**	**Std, Error**	**p-value**
Ω	0.0147	0.0828	0.0201**
α_1	0.0425	0.1878	0.0022**
β_1	0.0355	0.1208	0.0001***
β_2	0.365	0.690	0.0028*

Note: (*), (**) and (***) denote significance at 1%, 5% and 10% respectively.

table 13: the percent error of MSE and AMSE for the selected mode

Step ahead	MSA	AMSE
Ω1	0.002	0.489
10	0.591	0.985
20	0.631	0.625
35	0.732	0.639

Source: Authors' computation aided by R package v 4.1.1

This study has been established a modified GARCH for modelling and forecasting the stock exchange market evidence for Nigeria for period of 1985 to 2019. In table 4.1 the result reveal that there is stationarity at second difference because the ADF test statistic has more negative than the critical value and the p-value is less than 5% error i.e the unit root hypothesis is rejected. Also table 2 shows the the AIC and BIC values for the GARCH (p,q)-N at different sample sizes it reveals that GARCH(1,1)-N has the smallest information criterion for both AIC and BIC and all the coefficients are significant therefore, GARCH (1,1)-N is the best model. Also table 4.3 shows the ADF unit root test for GARCH (p,q)-GED from the results we conclude that the series is stationary at each selected sample. We reject the null hypothesis. In the same vain from table 4 shows the AIC and BIC values for GARCG (p,q)-GED from the results it shows GARCH (1,1)-GED has the smallest information criterion and also table 5 shoes the LM test and Ljung-Box Q2 are all significant we conclude that is the best model for forecasting.

Table 6 shows ADF unit root test for log-normal data, the result indicate that the data is stationary at each selected sample size. We reject the null hypothesis and conclude that the stationarity assumption is made. Also 7 shows the AIC and BIC values as its shows when the sample size increasing the information criterion values is decreasing, we conclude that GARCH(1,2)-LN has smallest information criterion value and its best for fitting and forecasting also the LM test and Ljung-Box Q2 of the selected order are significant from table 8. we conclude that order 1,1 and 1,2 are the best for the forecasting based on the information criterion and the configuration test.

Section 4.2 form table 9 shows the volatility fitting and forecasting of the simulated data with the selected order of the model. The MSE and the AMSE of the forecasted model shows that the GARCG (p,q)-LN has a minimum MSE and AMSE at each step ahead of the forecast therefore we conclude that the GARCH(p,q)-LN is the best for fitting and forecasting the stock market data.

Table 10 shows the stationarity test for the Nigerian stock market data. The results reveal that the data became stationary at second difference that is, the p-value is less than 5% and the test statistics has most negative therefore, we reject the null hypothesis and conclude that the stationarity assumption was made. Table 11 and 12 shows the estimated parameters for the selected model that is GARCH (p,q)-LN where p,q=1,2. it shows all the parameters of the model are significant at alpha level there for we do not reject the null hypothesis for all parameters. Also 13 shows the MSE and the AMSE for the forecasted data at different step ahead forecast and for each step the model has a minimum MSE and AMSE.

- **Conclusion**

The simulation results revealed that GARCH with non-normal distribution could further improve volatility forecasting performance and provide better forecast than the univariate GARCH model. The empirical results also confirm that considering the alternative error distribution in GARCH model with non-normal error distribution tend to outperform a model with normal error distribution therefore the non-normal error distribution such as Log-normal distribution in a GARCH model have demonstrated an ability to enhance the accuracy of volatility forecasting.

- **Recommendation**

Based on the above results, the following recommendations were made;

- Recommended how to determine order of GARCH model with the error term is not normally distributed.
- The GARCH with lognormal distribution tend to outer perform in fitting and forecasting the stock exchange data
- Extension of models such as EGARCH and TGARCH with non-normal distribution or other models to compare with can also be area of interest.

ACKNOWLEDGMENT

Gratitude goes to federal polytechnic Monguno for granting me the international based research fund (IBR) under TETfund with reference no TETF/DR&D/CE/POLY/BORNO/IBR/2024/VOL.1.

References

Amiratul, L. Mahmod, O. R.. S. Nazirah, R. Abdullah, H. and Pandian, V. (2020). "A novel fuzzy linear regression sliding window GARCH Model for Time Series". Applied Science 10(6) 1949.

Anoke Charity Ifeyiwa, Oda Stephen Idenyi, Chukwu Bishop Chibuzor and Agbi Promise E." testing the applicability of wagner's law in nigeria". volume 4-issue 6(2016) pp: 33-34

Banumathy, k and azhagaiah, R. (2015). “modelling stock market volatility: evidence from india.” managing global transitions 13 (1): 27–42.

Bekaert, G., Erb, C., Harver, C. and Viskanta, T. (1998) Distributional characteristics of emerging market returns and asset allocation. Journal of Portfolio Management, 102–116.

Bekaert, G. and Harvey, C. (1995) Emerging equity market volatility. Working paper, National Bureau of Economic Research (NBER).

Bollerslev T.P. (1986). Generalized Autoregressive Conditional Heteroscedasticity. Journal of Econometrics. 31(3), 307-327

Boris P, Davor H, Alexander M. Petersen. 2009 “Cross-correlation between volume change and price” 106(52) 22079-22084

Bollerslev& Wooldridge. 1992 “Quasi maximum likelihood estimation and inference in dynamic model with time varying covariances. 11(2): 143-172

Boris Podobnil, Davor Horvatic, Aleander m. Petersen and H. Eugene STANLEY (2009) “cross-correlation between volume change and price change” PNAS 106 (52) 22079-22084

Budd, Alan C., “GARCH (1,1) with sifted gamma distribution error” (2016). Electronic theses and dissertation. 1409

Cheteni, priviledge (2016) “stock market volatility using garch models: evidence from south Africaandchina stock markets.” mpra paper no. 77355

Cont, R. (2001) Empirical properties of asset returns: Stylized facts and statistical issues. Quantitative Finance, 1, 223–236.

Giorgio C, Roxana H and Alessandro P (2014) “Estimating GARCH-type models with systematicstable innovation: indirect inference maximum likelihood” URL:http://nbn-resolving.de/urn:nbn:de:bsz:352-290

Gokcan, S. (2000) Forecasting volatility of emerging stock markets: Linear versus non-linear GARCH models. Journal of Forecasting, 19, 499–504.

Gustav (2019) “impact of innovation, productivity and specialization on tourism competitiveness a spatial econometric analysis on European regions” current issues in tourism, 22:10, 1150-1169. DOI: 10.1080/13683500.2019.1366434.

Hung, J. C., Lee, M. C. and Liu, H. C. (2008) Estimation of Value-at-Risk for energy commodities via fat-tailed GARCH models. Energy Economics, 30, 1173–1191.

Khositkulporn, P (2013). “the factors affecting stock market volatility and contagion: thailand and south-east asia evidence.” Doctorate thesis, school of business, Victoria university, melbourne.

Khushnoor Khan, 2019 “Modeling stock markets volatility using GARCH models with Normal, Student’s t and stable Paretian distributions” stat. ISSN 2321-8916

Lingbing Feng &Yanlin Shi, 2017 “A simulation study on the distributions of disturbances in the GARCH model” issn 2332-2029, Liu, H.-C., Lee, Y.-H. and Lee, M.-C. (2009) Forecasting China stock markets volatility via GARCH models under skewed-GED distribution. Journal of Money, Investment and Banking, 7, 5–15.

Liu, H.-C. and Hung, J.-C. (2010) Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models. Expert Systems with Applications, 37, 4928–4934.

Murekachiro, D (2016). "time series volatility forecasting of the Zimbabwean stock exchange." the international journal of business & management, vol. 4 (3), pp: 41-52.

Usman, U., Musa Y. and Auwal H.M. (2018). Modeling Volatility of Nigeria Stock market return. MarketReturns using Garch models And Ranking MethodAn International Journal of Statisticsapplication and probability letters. J. Stat. Appl. Pro. Let. 5, No. 1, 13-24