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Fractional Mathematical Model for Simulating the Dynamics of Lassa Fever Virus in Nigeria With Control Strategies

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Abstract: *in this paper we extended mathematical models that were based on integer order derivatives to fractional order derivatives, we formulate and analyzed fractional mathematical modelling of dynamics of Lassa fever epidemic which includes both infected deceased and treatment compartments via Caputo sense. We proved that the propose mathematical model is biological and meaningfully well-posed. We also compute the basic reproduction number via next generation method. The Lassa fever-free equilibrium* ε_0 *is the only local asymptotic stable equilibrium if* $\mathcal{R}_{0h} < 1$ *and it is not stable when* $\mathcal{R}_{0h} > 1$. *Sensitivity analysis of the model parameters indicates that* ψ *and* β_3 *are the basic control parameters associated with persistence or eradication of Lassa fever virus. More precisely, there is an inverse relationship between* $\mathcal{R}_0$ *and* ψ. *In a similar note, increasing (decreasing) the value of* β_3 *keeping all other parameters fixed increases (decreases) the value of* $\mathcal{R}_0$. *We can infer from this result that good environmental sanitation and fumigation would reduce rodents' population thereby reducing the value of* β_3 *which leads to the decrease of* $\mathcal{R}_0$. *The existence and uniqueness of the solution of proposed FODE are established through the fixed-point theory. The numerical results and simulations of the extended fractional order mathematical model were explored in Caputo sense.*

Keywords: *Lassa fever virus; threshold parameters; model fitting; Sensitivity analysis; Control Parameters; Caputo fractional order derivative and fixed-point theory.*

1 Introduction

Lassa fever is also called Lassa hemorrhagic fever, is an infectious disease and a zoonotic viral illness instigated by the Lassa virus, a single-stranded RNA virus from the Arenaviridae family [1, 2]. The mastomys natalensis which known as a multimammate rat is the main host of this virus that is dominant in Sub-Saharan African as one of the most common rodent species [3–5]. the viral particle responsible for cause of Lassa fever was first identified in 1969 at Borno state northern region of Nigeria. However, the yearly estimated incidence in eastern and western regions of West Africa ranges from a hundred to three hundred thousand cases with nearly five thousand deaths [6–8] this momentum necessitated the Centers for Disease Control and Prevention (CDC) and World Health Organization (WHO) to declared Lassa fever as endemic and a health challenge in Western African Accordingly. The countries at the high-risk for Lassa fever (belt) include Liberia, Guinea, Sierra Leone, and Nigeria [6, 9–11]. the largest epidemic was reported to be in Nigeria, with report of many outbreaks from the aforementioned countries over the years. The largest outbreak of Lassa fever that swept through eighteen out of the thirty-six states of the country is reported to be in Nigeria, with over 400 confirmed cases were reported [12].

Although, the yearly increase of cases of Lassa virus has to do with various factor such as insufficient health facilities, polluted environment, and poor personal hygiene, to gather with the ecological climate factor rainfall and movement of harvested food into our communities. These activities are associated with an improve or increase in the host reservoir (mastomys rodents) to migrate from their natural habitation to the human environment, a reduce or diminished in the prevalence of Lassa fever is rely upon on human efforts in reducing the transmission proportion of this disease [3, 14].

Lassa fever has an incubation period between 6 and 21 days, hence, following this exposure period, infected humans are expected to start showing symptoms of the disease. Although about eighty percent of infected humans have only slight symptoms such as headaches, cough, muscle pain, sore throat, weakness, and fever. However, in severe cases, an infected human can develop more complications such as facial swelling, bleeding from the nose, respiratory distress, and low blood pressure [2, 11]. In a more critical situation, this disease can lead to death within fourteen days after the first appearance of the symptoms, due to neurological problems [2]. The Lassa virus is primarily spread to humans through human contact with food or substances that are contaminated by the urine or feces of an infected rodent [9], while secondary infection from human-to-human and laboratory transmissions are likewise possible [10]. Due to the absence of a vaccine against Lassa fever, prevention against infection has an important role in controlling the transmission of this disease in the population. Currently, since the eradication of mastomys rodent population is unrealistic, the present ways of avoiding the spread of this infection include the facilitation of good personal hygiene to avoid contact with infected rodents' secretions or excretions, and implementation of standard health facilities for effective testing, diagnosing and treatment of patients [10].

Literature revel that there is no confirmed cure or vaccine exists for Lassa fever yet, however, ribavirin is an antiviral drug that has been declared as an effective treatment for Lassa fever patients, if administered at the premature period of the infection [7, 9]. Consequently, the transmission dynamics of the virus is still not yet fully comprehended limited and far from being complete. Therefore, it is then important to urgently conduct various researches and explore new methods and techniques, which can help to better understanding of the outbreak process and controlling the spread of the virus.

Over the decade, mathematical models have become vital tools in studying the dynamics of diseases in a given population. The recent development of the use of mathematical models such as [2], has been developed for numerous diseases, to answer specific questions in an attempt to contribute to the understanding of the epidemiology of such disease under study. More specifically, studies have been carried out to further provide information on the transmission dynamics of Lassa fever (see [3, 7–9, 14]).

Mathematical model and simulation are a practical essential tool that helps us to improve our understanding of the real world [16]. It can help to determine the characteristics and magnitude of epidemic disease transmission, to predict its outbreak and to see which parameters are more influential in the dynamics of the disease

In recent decades, many physical problems have been modeled using the fractional calculus. The main reasons given for using fractional derivative models are that many systems show memory, history, or nonlocal effects, which can be difficult to model using integer order derivatives. The main reasons given for using fractional derivative models are that many

systems show memory, history, or nonlocal effects, which can be difficult to model using integer order derivatives. The basic theory and applications of fractional calculus and fractional differential equations can now be found in many studies (see, e.g., [15–19]). Although most of the early studies were based on the use of the Riemann–Liouville fractional order derivative or the Caputo fractional order derivative, it has been pointed out recently that these derivatives have the problem that their kernels have a singularity that occurs at the end point of an interval of definition. As a result, many new definitions of fractional derivatives have now been proposed in the literature (see, e.g., [20–28]). The fundamental differences among the fractional derivatives are their different kernels which can be selected to meet the requirements of different applications. For example, the main differences between the Caputo fractional derivative [16], the Caputo–Fabrizio derivative [22], and the Atangana– Baleanu fractional derivative [30] are that the Caputo derivative is defined using a power law, the Caputo–Fabrizio derivative is defined using an exponential decay law, and the Atangana–Baleanu derivative is defined using a Mittag–Leffler law. Examples of the applications of the new fractional operators to real world problems have been given in a number of recent papers. For example, Tateishi et al. [24] have compared the classical and new fractional time-derivatives in a study of anomalous diffusion. Also, Atangana have compared the Caputo–Fabrizio fractional derivative and the Atangana–Baleanu fractional derivative in modeling fractional delay differential equations [29] and in modeling chaotic systems [29]. They found that the power law derivative of the Riemann–Liouville fractional derivative or the Caputo–Fabrizio fractional derivative provides noisy information due to its specific memory properties. However, the Caputo–Fabrizio fractional derivative gives less noise than the power law one while the Atangana–Baleanu fractional derivative provides an excellent description.

Therefore, the novel feature of this research is to broaden existing knowledge by develop and formulates ordinary differential equations (integer-order derivatives) and extend it to the fractional differential equations (non integer-order derivatives) for the mathematical model of the dynamic of Lassa fever by incorporating treatment, environment contamination and infectious deceased population compartments, that will percipience the disease spread or control strategy, using Sensitivity analysis and numerical simulation of the model parameter values base on demographic data of Nigeria.

2 The Mathematical model description, formulation and analysis

2.1 model description

A mathematical model of Lassa fever virus by in cooperating infectious deceased compartment with treatment is introduced. The total human population $N_h(t)$ is divided into six compartments, namely, $S(t)$ represents the number of susceptible individuals, $E(t)$ represents the number of exposed individuals in the stage of Lassa fever virus infection, $I_1(t)$ represents the number of infected population, $I_2(t)$ represents the number of Infected deceased population (Individuals that contracted the disease through corpse or death infectious human), $T(t)$ represents the number of individuals being treated; $R(t)$ represents the number of individuals who have recovered and $N_r(t)$ is divided into two compartments, namely, $S_r(t)$ represents the number of Susceptible mastomys rats, $I_1(t)$ represents the number of Infected mastomys rats.

This study, engross on the effects of multiple transmission pathways of Lassa Fever concerning the progression of the infection in the human and rodent population. The use of multiple

transmission routes may give us a better understanding of the epidemiological structure of Lassa fever. Thus, the proposed model is formulated based on the following assumption.

(a) There is homogeneous mixing of members of the population under consideration.
(b) The dynamics outbreaks of Lassa fever in Africa (Nigeria) on yearly bases for relatively long period of time, allow a demographic process to take place as result of new additional inflow of new births and migration as well as deaths (natural or due to disease).
(c) Deceased individuals can still transmit the infection to susceptible populations before and during burial/funerals arrangement or ceremonies.
(d) Mostly poor resource countries are vulnerable to environmental transmission of the Lassa fever virus, due to the fact that environmental contribution is one of the essential factors enhancing the transmission process.
(e) Environmental contamination occurs when Mastomys rats shed the virus through urine or faeces. Consequently, direct contact with virus infested materials, through touching of soiled household objects, eating contaminated food, or exposure to open wounds or sores, can lead to infection, similarly infection may occur during rodents capture and grooming as Mastomys rats are sometimes consumed as a source of food.

Hence, the total human population and rodent's population are $N_h(t) = S(t) + E(t)+I_1(t)+I_2(t) + T(t) + R(t)$, and $N_r(t) = S_r(t)+I_r(t)$ respectively. Therefore, the pictorial diagrammatical representation of the model is shown in Fig. 1. According to Fig. 1, we have following model equations:

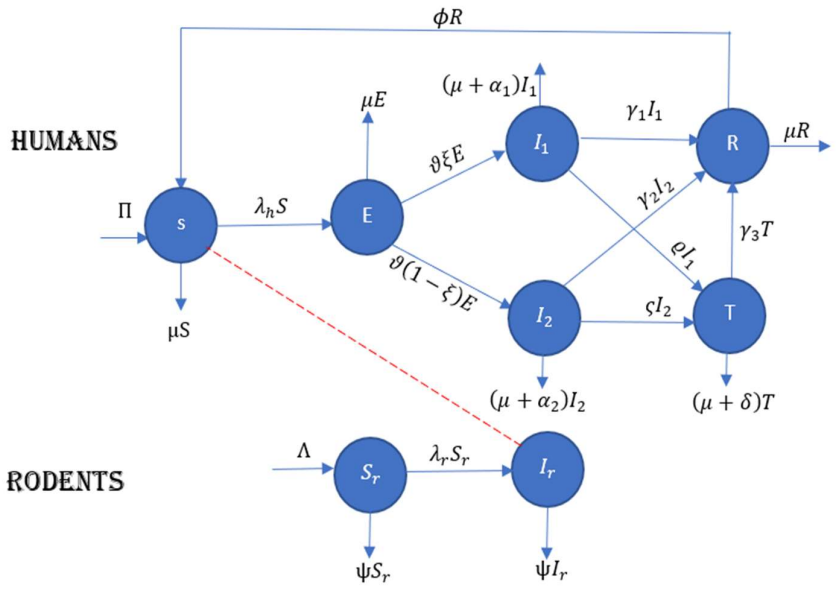

Figure 1. Flow Diagram of the Model 1

Description of the state variables and the parameters of the flow chart model

Symbol of the Variable	Description
$S(t)$	Susceptible human individuals
$E(t)$	Exposed human individuals
$I_1(t)$	Infected human individuals
$I_2(t)$	Infected deceased individuals n
$R(t)$	Recovery human individuals
$T(t)$	Treatment human individuals
$S_r(t)$	Susceptible mastomys rats
$I_r(t)$	Infected mastomys rats

Table 1. The state variables of the flow chart model system

Symbol of the Parameters	Descriptions
Π	Rate of recruitment of susceptible human population
Λ	Rate of recruitment of susceptible mastomys rat's population
μ , ψ	Natural death rate of human and mastomyns rat, respectively
ϕ	Rate at which immunity wanes after recovery
γ_1, γ_2 $and \gamma_3$	Recovery rate of infected, treated and infectious deceased human, respectively
α_1 and α_2	Disease induced death rates of individuals in $I_1(t)$ and $I_2(t)$, respectively
δ	Disease induced death rate of infected individuals in $T(t)$ compartment
$\varrho, \varsigma,$	Rates at which infected humans & infected deceased move to treatment class, respectively
$\xi \epsilon (0,1)$	Proportion of new exposed individual that become symptomatically infected
ϑ	Rate at which an exposed individual becomes infectious
φ	Transmission rates due to contact with infected deceased population (I_2) relative to the transmission rate due to infectious human corpse yet to be buried
β_1	The human-to-human contact rate
β_2	Mastomys rat-to-human contact rate
β_3	Mastomys rat-to-mastomys rat contact rate

Table 2. The parameters of the flow chart model system

2.2 Model Formulation

From the flow chart in figure 1. and the model assumptions, the following system of integer order Ordinary differential equation (ODEs) is the required model.

$$\frac{dS}{dt} = \Pi + \phi R - (\lambda_h + \mu)S$$

$$\frac{dE}{dt} = \lambda_h S - (\vartheta + \mu)E$$

$$\frac{dI_1}{dt} = \vartheta \xi E - (\gamma_1 + \varrho + \mu + \alpha_1)I_1$$

$$\frac{dI_2}{dt} = \vartheta(1-\xi)E - (\gamma_2 + \varsigma + \mu + \alpha_2)I_2 \qquad (1)$$

$$\frac{dT}{dt} = \varrho I_1 + \varsigma I_2 - (\gamma_3 + \mu + \delta)T$$

$$\frac{dR}{dt} = \gamma_1 I_1 + \gamma_2 I_2 + \gamma_3 T - (\phi + \mu)R$$

$$\frac{dS_r}{dt} = \Lambda - (\lambda_r + \psi)S_r$$

$$\frac{dI_r}{dt} = \lambda_r S_r - \psi I_r$$

With

$$\lambda_h = \frac{\beta_1(I_1 + \varphi I_2) + \beta_2 I_r}{N_h} \quad and \quad \lambda_r = \frac{\beta_3 I_r}{N_r} \tag{2}$$

where β_1 is the effective contact rate for human-to-human transmission. β_2 is a mastomys rat-to-human effective contact rate, and β_3 as mastomys rat-to-mastomys rat contact rate. Consequently, the parameters φ is a modification parameter which measures transmissibility reduction of infected deceased population (I_2).

2.3 Caputo–Fabrizio fractional model for transmission dynamics of Lassa fever

Many new definitions of fractional order derivatives have been proposed and used to develop and analyze mathematical models for a wide variety of real-life problems, and the advantages of memory, history, or nonlocal effects of fractional order derivatives motivated this research work. Consequently, Caputo and Fabrizio [22] recently developed a new fractional order derivative without any singularity in its kernel which accurately describe the memory effect in a real-life problem. The kernel of the new fractional derivative has the form of an exponential function. More recently, Losada and Nieto [23] derived the fractional integral associated with the new fractional Caputo–Fabrizio fractional derivative.

Now replacing the first-order time derivatives of the left-hand side of (1) by the fractional Caputo–Fabrizio derivative we obtain our new fractional derivative model (Caputo–Fabrizio fractional model for dynamics of Lassa fever epidemic mode) as follows:

$$CF_{D_t^\rho} S = \Pi + \phi R - (\lambda_h + \mu)S$$

$$CF_{D_t^\rho} E = \lambda_h S - (\vartheta + \mu)E$$

$$CF_{D_t^\rho} I_1 = \vartheta\xi E - (\gamma_1 + \varrho + \mu + \alpha_1)I_1$$

$$CF_{D_t^\rho} I_2 = \vartheta(1 - \xi)E - (\gamma_2 + \varsigma + \mu + \alpha_2)I_2$$

$$CF_{D_t^\rho} T = \varrho I_1 + \varsigma I_2 - (\gamma_3 + \mu + \delta)T \tag{3}$$

$$CF_{D_t^\rho} R = \gamma_1 I_1 + \gamma_2 I_2 + \gamma_3 T - (\phi + \mu)R$$

$$CF_{D_t^\rho} S_r = \Lambda - (\lambda_r + \psi)S_r$$

$$CF_{D_t^\rho} I_r = \lambda_r S_r - \psi I_r$$

where $CF_{D_t}^{\rho}$ represents the Caputo-Fabrizio fractional derivative of order $0 < \rho i \leq 1$, with the non-negative initial conditions

$$S(0) = S_0, E(0) = E_0, I_1(0) = I_{1_0}, I_2(0) = I_{2_0}, T(0) = T_0, R(0) = R_0\ , T(0) = S_{r_0}, I_r(0) = I_{r_0} \quad (4)$$

We assume that the fractional orders $(0 < \rho i < 1,\ i = 1, 2, \ldots, 8)$ for each of the eight populations can be different.

3 Positivity and Boundedness

To retain the biological validity of model 1, we must prove that the solutions of the fractional-order system (3) with the force of infection are positive and bounded for $t > 0$. Since the system monitors humans' and rodents' populations, all associated parameters are nonnegative. We will now prove the positivity and boundedness of the solutions to ensure the system is mathematically well-posed and biologically meaningful.

Theorem1. *Let the initial coditions* $S(0) > 0, E(0) > 0, I_1(0) > 0, I_2(0) > 0, T(0) > 0,$ $R(0) > 0, S_r(0) > 0,$ *and* $I_r(0) > 0,$ *then* $S(t), E(t),\ I_1(t),\ I_2(t), T(t),\ R(t),\ S_r(t)$ *and* $I_r(t)$ *of the model* (1) *are positive for all* $t \geq 0$.

Proof. Suppose $S(t)$ is not positive, then there exists a first time, say $t^* > 0$, such that $S(t) > 0$ for all $t \in [0, t^*)$ and $S(t^*) = 0$. By inspection of the equation of $E(t)$, we have that

$$CF_{D_t}^{\rho} E \geq -(\vartheta + \mu)\ E(t), for\ \ t \in [0, t^*),$$

Hence, it follows that,

$$E > 0\ for\ t \in [0, t^*).$$

Thus, it is clear from the first equation of model 1 that

$$CF_{D_t}^{\rho} S \geq -(\lambda_h + \mu) S(t), for\ t \in [0, t^*).$$

It follows that $S(t^*) > 0$ which contradicts $S(t^*) = 0$. therefore, $S(t)$ is positive. Using similar approach as that for $S(t)$, it is easy to show that $E(t) > 0,\ I_1(t) > 0,\ I_2(t) > 0, T(t) > 0,$ $R(t) > 0,\ S_r(t) > 0\ and\ \ I_r(t) > 0$. Hence the proof.

3.1 Invariant Region

In order to retain the biological feasible region of fractional-order system (3) we will be analysed in a biologically feasible region as follows. Consider the biologically feasible region consisting of

$\Omega = \Omega_h \times \Omega_r \in \mathbb{R}_+^6 \times \mathbb{R}_+^2$ with

$\Omega_h = \{S, E,\ I_1,\ I_2, T, R\ \in \mathbb{R}_+^6 : N_h \leq \frac{\Pi}{\mu}\ \}$

and

$\Omega_r = \{\ S_r,\ I_r, \in \mathbb{R}_+^2 : N_r \leq \frac{\Lambda}{\psi}\}$

It can be shown that the set Ω is a positively invariant set and global attractor of this system. This implies that any phase trajectory initiated anywhere in the nonnegative region $\mathbb{R}_+^8$ enters the feasible region Ω and remains in thereafter.

Lemma 1. The biological feasible region $\Omega = \Omega_h \cup \Omega_r \subset \mathbb{R}_+^6 \times \mathbb{R}_+^2$ of the Lassa fever model (1) is positively invariant with nonnegative initial conditions in $\mathbb{R}_+^8$.

Proof The following steps are followed to establish the positive invariance of Ω (i.e., solutions in Ω remain in Ω for all $t > 0$). The rate of change of the total human and rodent populations $N_h$and N_r respectively, are obtained by adding the respective components of fractional-order system (3) which result to

$$CF_{D_t^\rho} N_h(t) = \Pi - \mu N_h(t) - \{(\alpha_1) I_1(t) + (\alpha_2) I_2(t) + (\delta) T(t)\} \; and$$

$$CF_{D_t^\rho} N_r(t) = \Lambda - \psi N_r(t)$$

so that,

$$CF_{D_t^\rho} N_h(t) \leq \Pi - \mu N_h(t) \; and \; CF_{D_t^\rho} N_r(t) \leq \Lambda - \psi N_r(t) \qquad (5)$$

Hence, $N_h(t) \leq \mu N_h(0) e^{\mu t} + \frac{\Pi}{\mu}(1 - e^{-\mu t})$ and $N_r(t) \leq \psi N_r(0) e^{\psi t} + \frac{\Lambda}{\psi}\left(1 - e^{-\psi t}\right)$.

In particular, $N_h(t) \leq \frac{\Pi}{\mu}$ and $N_r(t) \leq + \frac{\Lambda}{\psi}$ if the total human and rodent population at the initial instant of time, $N_h(0) \leq \frac{\Pi}{\mu}$ and $N_r(0) \leq + \frac{\Lambda}{\psi}$, respectively. So, the region Ω is positively invariant. Thus, it is consequently adequate to consider the dynamics of Lassa fever governed by fractional-order system (3) in the biological feasible region Ω , where the model is considered to be epidemiologically and mathematically well posed.

3.2 Existence and Stability of Lassa fever free equilibrium

The Lassa fever-free equilibrium of the fractional-order system (3) is obtained by finding the steady-state solution in the absence of Lassa fever infection. This involves setting the right-hand side of equation (3) to zero and solving the resulting algebraic equations simultaneously. We have

$$CF_{D_t^\rho} S = \Pi - \mu S$$

$$CF_{D_t^\rho} S_r = \Lambda - \psi S_r$$

And the the disease-free equilibrium state denoted by $\mathcal{E}_0$ is

$$\mathcal{E}_0 = (S^*, E^*, {I_1}^*, {I_2}^*, T^*, R^*, S_r^*, I_r^*) = \left(\frac{\Pi}{\mu}, 0,0,0,0,0, \frac{\Lambda}{\psi}, 0\right) \qquad (6)$$

3.3 Basic reproduction number

The next-generation matrix method is used on system (3) for determining the reproduction number $\mathcal{R}_0$. The epidemiological quantity $\mathcal{R}_0$, called the reproduction number, measures the typical number of Lassa fever cases that a Lassa fever-infected individual can generate in a human population that is completely susceptible.

Using the notation in [35], the matrices F and V for the new infection teams and the remaining transfer or transition teams are as follows

$$\text{Gain} \Rightarrow F = \begin{pmatrix} 0 & \beta_1 & \beta_1\varphi & \beta_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 \end{pmatrix}$$

$$\text{Losses} \Rightarrow V = \begin{pmatrix} \vartheta+\mu & 0 & 0 & 0 \\ -\vartheta\xi & \mu+\alpha_1+\gamma_1+\varrho & 0 & 0 \\ -\vartheta(1-\xi) & 0 & \mu+\alpha_2+\gamma_2+\varsigma & 0 \\ 0 & 0 & 0 & \psi \end{pmatrix}$$

Thus, the basic reproduction $\mathcal{R}_0$ of the model (3) is the spectral radius of the next-generation matrix FV^{-1}. It follows then that the associated reproduction number, denoted by $\mathcal{R}_0$, is given by

$$\mathcal{R}_0 = \mathcal{R}_{0h} + \mathcal{R}_{0r} = \frac{Q_1Q_2Q_4\beta_3 + Q_2Q_3\varphi\psi\beta_1 + Q_4\vartheta\xi\psi\beta_1}{Q_1Q_2Q_4\psi} \tag{7}$$

Where

$$\mathcal{R}_{0h} = \frac{\beta_1(Q_2Q_3\varphi + Q_4\vartheta\xi)}{Q_1Q_2Q_4} \tag{8}$$

$$\mathcal{R}_{0r} = \frac{\beta_3}{\psi} \tag{9}$$

With $Q_1 = (\vartheta+\mu),\ Q_2 = (\mu+\alpha_1+\gamma_1+\varrho), Q_3 = \vartheta-\vartheta\xi,\ Q_4 = (\mu+\alpha_2+\gamma_2+\varsigma)\ and\ Q_5 = (\mu+\delta+\gamma_3)$ (10)

The threshold quantity $\mathcal{R}_0$ given in (7) defined the quantity $\mathcal{R}_{0h}$ (the basic reproduction number of human population) and $\mathcal{R}_{0r}$ (the basic reproduction number of rodent population) which measures the contribution of Lassa fever risk caused by human and rodent population respectively, it can be observed that the rise in any of the threshold quantity results in the high risk of Lassa fever in the population.

4 Existence and uniqueness of solutions of the model

Examine the existence and uniqueness of the solutions of the Caputo– Fabrizio fractional model for dynamics of Lass fever epidemic in Eq. (3) with initial conditions (4). Using fixed point theory [33, 34], we can prove the existence of solutions for the model as follows

Applying the Caputo–Fabrizio fractional integral operator in both sides of Eq. (3), we have

$$S(t) - S(0) = CF_{I_t^{\rho1}}[\Pi + \phi R - (\lambda_h+\mu)S],$$

$$E(t) - E(0) = CF_{I_t^{\rho2}}[\lambda_h S - (\vartheta+\mu)E],$$

$$I_1(t) - I_1(0) = CF_{I_t^{\rho3}}[\,\vartheta\xi E - (\gamma_1+\varrho+\mu+\alpha_1)I_1],$$

$$I_2(t) - I_2(0) = CF_{I_t}^{\rho 4}[\vartheta(1-\xi)E - (\gamma_2 + \varsigma + \mu + \alpha_2)I_2],$$

$$T(t) - T(0) = CF_{I_t}^{\rho 5}[\varrho I_1 + \varsigma I_2 - (\gamma_3 + \mu + \delta)T\ ], \tag{11}$$

$$R(t) - R(0) = CF_{I_t}^{\rho 6}[\gamma_1 I_1 + \gamma_2 I_2 + \gamma_3 T - (\phi + \mu)R]$$

$$S_r(t) - S_r(0) = CF_{I_t}^{\rho 7}[\ \Lambda - (\lambda_r + \psi)S_r],$$

$$I_r(t) - I_r(0) = CF_{I_t}^{\rho 8}[\lambda_r S_r - \psi I_r],$$

Then, the kernels of the model system can be written as follows

$$K_1(t,s) = \Pi + \phi R - (\lambda_h + \mu)S\ ,$$

$$K_2(t,E) = \ \lambda_h S(t) - (\vartheta + \mu)E(t),$$

$$K_3(t,I_1) = \ \vartheta\xi E(t) - (\gamma_1 + \varrho + \mu + \alpha_1)I_1(t),$$

$$K_4(t,I_2) = \ \vartheta(1-\xi)E(t) - (\gamma_2 + \varsigma + \mu + \alpha_2)I_2(t), \tag{12}$$

$$K_5(t,T) = \varrho I_1(t) + \varsigma I_2(t) - (\gamma_3 + \mu + \delta)T(t),$$

$$K_6(t,R) = \ \gamma_1 I_1(t) + \gamma_2 I_2(t) + \gamma_3 T(t) - (\phi + \mu)R(t),$$

$$K_7(t,S_r) = \ \Lambda - (\lambda_r + \psi)S_r(t),$$

$$K_8(t,I_r) = \lambda_r S_r(t) - \psi I_r(t),$$

and the functions

$$\Lambda(\rho) = \frac{2(1-\rho)}{(2-\rho)M(\rho)}\ \ and\ \ \Delta(\rho) = \frac{2\rho}{(2-\rho)M(\rho)} \tag{13}$$

In proving the following theorems, we will assume that $S, E,\ I_1,\ I_2, T, R$, and $\ S_r,\ I_r$ are nonnegative bounded functions,

i.e.,$\|S(t)\| \le \theta_1, \|E(t)\| \le \theta_2, \|\ I_1(t)\| \le \theta_3, \|\ I_2(t)\| \le \theta_4, \|T(t)\| \le \theta_5, \|R(t)\| \le \theta_6, \|\ S_r(t)\| \le \theta_7 and\ \ \|\ I_r(t)\| \le\ \theta_8$

where $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7\ and\ \theta_8$ are some positive constants. Denote

$$\aleph_1 = \lambda_h + \mu, \aleph_2 = \vartheta + \mu,\ \aleph_3 = \mu + \gamma, \aleph_4 = \gamma_1 + \varrho + \mu + \alpha_1, \aleph_5 = \gamma_2 + \varsigma + \mu + \alpha_2,\ \aleph_6 = \mu + \gamma, \aleph_7 = \psi\ and\ \aleph_8 = \lambda_r + \psi, \tag{14}$$

Applying the definition of the Caputo–Fabrizio fractional integral in Eq. (11), we obtain.

$$S(t) - S(0) = \Lambda(\rho_1)K_1(t,S) + \Delta(\rho_1)\int_0^t K_1(y,S)dy,$$

$$E(t) - E(0) = \Lambda(\rho_2)K_2(t,E) + \Delta(\rho_2)\int_0^t K_2(y,E)dy,$$

$$I_1(t) - I_1(0) = \Lambda(\rho_3)K_3(t,I_1) + \Delta(\rho_3)\int_0^t K_3(y,I_1)dy,$$

$$I_2(t) - I_2(0) = \Lambda(\rho_4)K_4(t,I_2) + \Delta(\rho_4)\int_0^t K_4(y,I_2)dy,$$

$$T(t) - T(0) = \Lambda(\rho_5)K_5(t,T) + \Delta(\rho_5)\int_0^t K_5(y,T)dy, \tag{15}$$

$$R(t) - R(0) = \Lambda(\rho_6)K_6(t,R) + \Delta(\rho_6)\int_0^t K_6(y,R)dy$$

$$S_r(t) - S_r(0) = \Lambda(\rho_7)K_7(t,S_r) + \Delta(\rho_7)\int_0^t K_7(y,S_r)dy,$$

$$I_r(t) - I_r(0) = \Lambda(\rho_8)K_8(t,I_r) + \Delta(\rho_8)\int_0^t K_8(y,I_r)dy,$$

Theorem1:If the following inequality holds $0 \leq M = \max\{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7, \aleph_8\} < 1(16)$

then the kernels $K_1, K_2, K_3, K_4, K_5, K_6, K_7 \; and \; K_8$ satisfy Lipschitz conditions and are contraction mappings.

Proof. We consider the kernel K_1. Let S and S_1 be any two functions, then we have

$$\|K_1(t,S) - K_1(t,S_I)\| = \|-\lambda_h(S(t) - S_1(t)) - \mu(S(t) - S_1(t))\| \quad (17)$$

Using the triangle inequality for norms on the right-hand side of Eq. (14), we obtain

$$\|K_1(t,S) - K_1(t,S_I)\| \leq \|-\lambda_h(S(t) - S_1(t))\| + \|\mu(S(t) - S_1(t))\| \leq (\lambda_h + \mu)\|S(t) - S_1(t)\| \leq (\lambda_h + \mu)\|S(t) - S_1(t)\| = \aleph_1\|S(t) - S_1(t)\|. \quad (18)$$

Where $\aleph_1$ is defined in Eq. (12). Similar results for the kernels $K_2, K_3, K_4, K_5, K_6, K_7 \; and \; K_8$ can be obtained using $\{E, E_1\}$,$\{I_1, I_{1_1}\}$, $\{I_2, I_{2_1}\}$,$\{T, T_1\}$, $\{R, R_1\}$,$\{S_r, S_{r_1}\} and \; \{I_r, I_{r_1}\}$, respectively, as follows:

$$\|K_2(t,E) - K_2(t,E_I)\| \leq \aleph_2\|I(t) - I_1(t)\|$$

$$\|K_3(t,I_1) - K_3(t,I_{1_I})\| \leq \aleph_3\|I_1(t) - I_{1_1}(t)\|$$

$$\|K_4(t,I_2) - K_4(t,I_{2_I})\| \leq \aleph_4\|I_2(t) - I_2(t)\|$$

B$\|K_5(t,T) - K_5(t,T_I)\| \leq \aleph_5\|T(t) - T_1(t)\|$

$$\|K_6(t,R) - K_6(t,R_I)\| \leq \aleph_6\|R(t) - R_1(t)\| \quad (19)$$

$$\|K_7(t,S_r) - K_7(t,S_{r_I})\| \leq \aleph_7\|S_r(t) - S_{r_1}(t)\|$$

$$\|K_8(t,V) - K_8(t,I_{r_I})\| \leq \aleph_8\|S_r(t) - S_{r_1}(t)\|$$

where $\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7 \; and \; \aleph_8$ are defined in Eq. (12). Therefore, the Lipschitz conditions are satisfied for $K_2, K_3, K_4, K_5, K_6, K_7 \; and \; K_8$. In addition, since $0 \leq M = \max\{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7, \aleph_8\} < 1$, the kernels are contractions. From Eq. (15), the state variables can be displayed in terms of the kernels as follows:

$$S(t) = S(0) + \Lambda(\rho_1)K_1(t,S) + \Delta(\rho_1)\int_0^t K_1(y,S)dy,$$

$$E(t) = E(0) + \Lambda(\rho_2)K_2(t,E) + \Delta(\rho_2)\int_0^t K_2(y,E)dy,$$

$$I_1(t) = I_1(0) + \Lambda(\rho_3)K_3(t,I_1) + \Delta(\rho_3)\int_0^t K_3(y,I_1)dy,$$

$$I_2(t) = I_2(0) + \Lambda(\rho_4)K_4(t,I_2) + \Delta(\rho_4)\int_0^t K_4(y,I_2)dy,$$

$$T(t) = T(0) + \Lambda(\rho_5)K_5(t,T) + \Delta(\rho_5)\int_0^t K_5(y,T)dy, \tag{20}$$

$$R(t) = R(0) + \Lambda(\rho_6)K_6(t,R) + \Delta(\rho_6)\int_0^t K_6(y,R)dy$$

$$S_r(t) = S_r(0) + \Lambda(\rho_7)K_7(t,S_r) + \Delta(\rho_7)\int_0^t K_7(y,S_r)dy,$$

$$I_r(t) = I_r(0) + \Lambda(\rho_8)K_8(t,I_r) + \Delta(\rho_8)\int_0^t K_8(y,I_r)dy,$$

Using Eq. (20), we now introduce the following recursive formulas:

$$S_n(t) = \Lambda(\rho_1)K_1(t,S_{n-1}) + \Delta(\rho_1)\int_0^t K_1(y,S_{n-1})dy,$$

$$E_n(t) = \Lambda(\rho_2)K_2(t,E_{n-1}) + \Delta(\rho_2)\int_0^t K_2(y,E_{n-1})dy,$$

$$I_{n1}(t) = \Lambda(\rho_3)K_3\big(t,I_{(n-1)1}\big) + \Delta(\rho_3)\int_0^t K_3\big(y,I_{(n-1)1}\big)dy,$$

$$I_{n2}(t) = \Lambda(\rho_4)K_4\big(t,I_{(n-1)2}\big) + \Delta(\rho_4)\int_0^t K_4\big(y,I_{(n-1)2}\big)dy, \tag{21}$$

$$T_n(t) = \Lambda(\rho_5)K_5(t,T_{n-1}) + \Delta(\rho_5)\int_0^t K_5(y,T_{n-1})dy,$$

$$R_n(t) = \Lambda(\rho_6)K_6(t,R_{n-1}) + \Delta(\rho_6)\int_0^t K_6(y,R_{n-1})dy$$

$$S_{nr}(t) = \Lambda(\rho_7)K_7\big(t,S_{(n-1)r}\big) + \Delta(\rho_7)\int_0^t K_7\big(y,S_{(n-1)r}\big)dy,$$

$$I_{nr}(t) = \Lambda(\rho_8)K_8\big(t,I_{(n-1)r}\big) + \Delta(\rho_8)\int_0^t K_8\big(y,I_{(n-1)r}\big)dy,$$

The initial components of the above recursive formulas are determined by the given initial conditions as follows:

$$S_0(t) = S(0), E_0(t) = E(0), I_{1_0}(t) = I_1(0), I_{2_0}(t) = I_2(0)$$
$$T_0(t) = T(0), R_0(t) = R(0), S_{r_0}(t) = S_r(0), I_{r_0}(t) = I_r(0) \tag{22}$$

The differences between the consecutive terms for the recursive formulas can be written as

$$\begin{aligned}\Phi_{1n}(t) &= S_n(t) - S_{n-1}(t)\\ &= \Lambda(\rho_1)(K_1(t,S_{n-1}) - K_1(t,S_{n-2}))\\ &\quad + \Delta(\rho_1)\int_0^t (K_1\big(y,S_{n-1} - K_1(t,S_{n-2})\big)dy,\end{aligned}$$

$$\begin{aligned}\Phi_{2n}(t) &= E_n(t) - E_{n-1}(t)\\ &= \Lambda(\rho_2)(K_2(t,E_{n-1}) - K_2(t,E_{n-2}))\\ &\quad + \Delta(\rho_2)\int_0^t (K_2\big(y,E_{n-1} - K_2(t,E_{n-2})\big)dy\end{aligned} \tag{23}$$

$$\begin{aligned}\Phi_{3n}(t) &= I_{1_n}(t) - I_{1_{n-1}}\\ &= \Lambda(\rho_3)(K_3\big(t,I_{1_{n-1}}\big) - K_3\big(t,I_{1_{n-2}}\big)) + \Delta(\rho_3)\int_0^t (K_3\left(y,I_{1_{n-1}} - K_3\big(t,I_{1_{n-2}}\big)\right)dy,\end{aligned}$$

$$\begin{aligned}\Phi_{4n}(t) &= I_{2_n}(t) - I_{2_{n-1}}(t)\\ &= \Lambda(\rho_4)(K_4\big(t,I_{2_{n-1}}\big) - K_4\big(t,I_{2_{n-2}}\big)) + \Delta(\rho_4)\int_0^t (K_4\left(y,I_{2_{n-1}} - K_4\big(t,I_{2_{n-2}}\big)\right)dy,\end{aligned}$$

$$\Phi_{5n}(t) = T_n(t) - T_{n-1}(t)$$
$$= \Lambda(\rho_5)(K_5(t, T_{n-1}) - K_5(t, T_{n-2}))$$
$$+ \Delta(\rho_5)\int_0^t (K_5(y, T_{n-1} - K_5(t, T_{n-2}))dy,$$

$$\Phi_{6n}(t) = R(t) - R_{n-1}(t)$$
$$= \Lambda(\rho_6)(K_6(t, R_{n-1}) - K_6(t, R_{n-2}))$$
$$+ \Delta(\rho_6)\int_0^t (K_6(y, R_{n-1} - K_6(t, R_{n-2}))dy,$$

$$\Phi_{7n}(t) = S_{r_n}(t) - S_{r_{n-1}}(t)$$
$$= \Lambda(\rho_7)(K_7(t, S_{r_{n-1}}) - K_7(t, S_{r_{n-2}}))$$
$$+ \Delta(\rho_7)\int_0^t (K_7\left(y, S_{r_{n-1}} - K_7(t, S_{r_{n-2}})\right)dy,$$

$$\Phi_{8n}(t) = I_{r_n}(t) - I_{r_{n-1}}(t)$$
$$= \Lambda(\rho_8)(K_8(t, I_{r_{n-1}}) - K_8(t, I_{r_{n-2}})) + \Delta(\rho_8)\int_0^t (K_8\left(y, I_{r_{n-1}} - K_8(t, I_{r_{n-2}})\right)dy,$$

For $S_n(t) = \sum_{i=1}^n \Phi_{1i}(t), E_n(t) = \sum_{i=1}^n \Phi_{2i}(t), I_{1_n}(t) = \sum_{i=1}^n \Phi_{3i}(t), I_{2_n}(t) = \sum_{i=1}^n \Phi_{4i}(t)$ $, T_n(t) = \sum_{i=1}^n \Phi_{5n_i}(t), R_n(t) = \sum_{i=1}^n \Phi_{6i}(t), S_{r_n}(t) = \sum_{i=1}^n \Phi_{7i}(t), I_{r_n}(t) = \sum_{i=1}^n \Phi_{8i}(t)$ (24)

Now let generate the recursive inequalities for the differences $\Phi_{1n}, \Phi_{2n}, \Phi_{3n}, \Phi_{4n}, \Phi_{5n}, \Phi_{6n}, \Phi_{7n}\ and\ \Phi_{8n}$ as follows

$$\|\Phi_{1n}(t)\| = \|S_n(t) - S_{n-1}(t)\|$$
$$= \Big\|\Lambda(\rho_1)(K_1(t, S_{n-1}) - K_1(t, S_{n-2}))$$
$$+ \Delta(\rho_1)\int_0^t (K_1(y, S_{n-1}$$
$$- K_1(t, S_{n-2}))dy\Big\| \qquad (25)$$

Using t triangle inequality for norms to Eq. (25), we have

$\|S_n(t) - S_{n-1}(t)\| = \|\Lambda(\rho_1) \| K_1(t, S_{n-1}) - K_1(t, S_{n-2})\| + \Delta(\rho_1)\int_0^t \|K_1(y, S_{n-1}) - K_1(t, S_{n-2})\|dy$

Then, since the kernel K_1 satisfies the Lipschitz condition with Lipschitz constant $\aleph_1$, we have

$$\|S_n(t) - S_{n-1}(t)\| \leq \|\Lambda(\rho_1)\aleph_1 \| S_{n-1} - S_{n-2}\| + \Delta(\rho_1)\aleph_1 \int_0^t \|S_{n-1} - S_{n-2}\|dy \qquad (26)$$

therefore, we have

$$\|\Phi_{1n}(t)\| \leq \Lambda(\rho_1)\aleph_1\big\|\Phi_{1(n-1)}(t)\big\| + \Delta(\rho_1)\aleph_1 \int_0^t \big\|\Phi_{1(n-1)}(y)\big\|dy \qquad (27)$$

Following the same procedures, we have

$$\|\Phi_{2n}(t)\| \leq \Lambda(\rho_2)\aleph_2\|\Phi_{2(n-1)}(t)\| + \Delta(\rho_2)\aleph_2\int_0^t\|\Phi_{2(n-1)}(y)\|dy$$

$$\|\Phi_{3n}(t)\| \leq \Lambda(\rho_3)\aleph_3\|\Phi_{3(n-1)}(t)\| + \Delta(\rho_3)\aleph_3\int_0^t\|\Phi_{3(n-1)}(y)\|dy$$

(28)

$$\|\Phi_{4n}(t)\| \leq \Lambda(\rho_4)\aleph_4\|\Phi_{4(n-1)}(t)\| + \Delta(\rho_4)\aleph_4\int_0^t\|\Phi_{4(n-1)}(y)\|dy$$

$$\|\Phi_{5n}(t)\| \leq \Lambda(\rho_5)\aleph_5\|\Phi_{5(n-1)}(t)\| + \Delta(\rho_5)\aleph_5\int_0^t\|\Phi_{5(n-1)}(y)\|dy$$

$$\|\Phi_{6n}(t)\| \leq \Lambda(\rho_6)\aleph_6\|\Phi_{6(n-1)}(t)\| + \Delta(\rho_6)\aleph_6\int_0^t\|\Phi_{6(n-1)}(y)\|dy$$

$$\|\Phi_{7n}(t)\| \leq \Lambda(\rho_7)\aleph_7\|\Phi_{7(n-1)}(t)\| + \Delta(\rho_7)\aleph_7\int_0^t\|\Phi_{7(n-1)}(y)\|dy$$

$$\|\Phi_{8n}(t)\| \leq \Lambda(\rho_8)\aleph_8\|\Phi_{8(n-1)}(t)\| + \Delta(\rho_8)\aleph_8\int_0^t\|\Phi_{8(n-1)}(y)\|dy$$

Theorem 5. If there exists a time $t_0 > 0$ such that the following inequalities hold:$\Lambda(\rho_1)\aleph_1 + \Delta(\rho_1)\aleph_1 t_0 > 1, \quad for\ i = 1,2,\dots,8,$ (29)

then a system of solutions exists for the fractional Lassa fever model (3)– (4).

Proof. Since the functions $S(t), E(t), I_1(t), I_2(t), T(t), R(t), S_r(t)\ and\ I_r(t)$ are assumed to be bounded and each of the kernels satisfies a Lipschitz condition, the following relations can be obtained.

Using Eqs. (27)– (28) recursively:

$$\|\Phi_{1n}(t)\| \leq \|S(0)\|[\Lambda(\rho_1)\aleph_1 + \Delta(\rho_1)\aleph_1]^n$$

$$\|\Phi_{2n}(t)\| \leq \|E(0)\|\,[\Lambda(\rho_2)\aleph_2 + \Delta(\rho_2)\aleph_2]^n$$

$$\|\Phi_{3n}(t)\| \leq \|\,I_1(0)\|[\Lambda(\rho_3)\aleph_3 + \Delta(\rho_3)\aleph_3]^n \quad (30)$$

$$\|\Phi_{4n}(t)\| \leq \|\,I_2(0)\|[\Lambda(\rho_4)\aleph_4 + \Delta(\rho_4)\aleph_4]^n$$

$$\|\Phi_{5n}(t)\| \leq \|T(0)\|[\Lambda(\rho_5)\aleph_5 + \Delta(\rho_5)\aleph_5]^n$$

$$\|\Phi_{6n}(t)\| \leq \|R(0)\|[\Lambda(\rho_6)\aleph_6 + \Delta(\rho_6)\aleph_6]^n$$

$$\|\Phi_{7n}(t)\| \leq \|\,S_r(0)\|[\Lambda(\rho_7)\aleph_7 + \Delta(\rho_7)\aleph_7]^n$$

$$\|\Phi_{8n}(t)\| \leq \|\,I_r(0)\|[\Lambda(\rho_8)\aleph_8 + \Delta(\rho_8)\aleph_8]^n$$

Equation (30) shows the existence and smoothness of the functions defined in Eq. (25). To complete the proof, we prove that the functions $S_n(t), E_n(t), I_{1_n}(t), I_{2_n}(t), T_n(t), R_n(t), S_r(t)\ and\ I_r(t)$ converge to a system of solutions of (3)– (4). We introduce $B_n(t), C_n(t), E_n(t), F_n(t), G_n(t), H(t), U_n(t)\ and\ W_n(t)$, as the remainder terms after n iterations, i.e.,

$$S(t) - S(0) = S_n(t) - B_n(t),$$

$$E(t) - E(0) = E_n(t) - C_n(t),$$

$$I_1(t) - I_1(0) = I_{1_n}(t) - E_n(t),$$

$$I_2(t) - I_2(0) = I_{2_n}(t) - F_n(t),$$

$$T(t) - T(0) = T_n(t) - G_n(t),, \tag{31}$$

$$R(t) - R(0) = R_n(t) - H_n(t),$$

$$S_r(t) - S_r(0) = S_{r_n}(t) - U_n(t),$$

$$I_r(t) - I_r(0) = I_{r_n}(t) - W_n(t),$$

Then, using the triangle inequality and the Lipschitz condition for K_1, we have

$\|B_n(t)\| = \left\|\Lambda(\rho_1)(K_1(t,S) - K_1(t,S_{n-1})) + \Delta(\rho_1)\int_0^t (K_1(y,S) - K_1(y,S_{n-1}))dy\right\| \leq \Lambda(\rho_1)\|(K_1(t,S) - K_1(t,S_{n-1})\| + \Delta(\rho_1)\int_0^t \|K_1(y,S) - K_1(y,S_{n-1})\|dy \leq \Lambda(\rho_1)\aleph_1\|S - S_{n-1}\| + \Delta(\rho_1)\aleph_1 B_n(t)\|S - S_{n-1}\|t.$

Repeating same process, we have;

$$\|B_n(t)\| \leq [(\Lambda(\rho_1) + \Delta(\rho_1)t)\aleph_1]^{n+1}\theta_1 \tag{32}$$

At t_0 we have $\|B_n(t)\| \leq [(\Lambda(\rho_1) + \Delta(\rho_1)t_0)\aleph_1]^{n+1}\theta_1$ (33)

Taking the limit on Eq. (33) as $n \to \infty$ and then using condition (29), we obtain$\|B_n(t)\| \to 0$. Using the same process as described above, we have the following relations:

$$\|C_n(t)\| \leq [(\Lambda(\rho_2) + \Delta(\rho_2)t_0)\aleph_2]^{n+1}\theta_2 \tag{34}$$

$$\|E_n(t)\| \leq [(\Lambda(\rho_3) + \Delta(\rho_3)t_0)\aleph_3]^{n+1}\theta_3 \tag{35}$$

$$\|F_n(t)\| \leq [(\Lambda(\rho_4) + \Delta(\rho_4)t_0)\aleph_4]^{n+1}\theta_4 \tag{36}$$

$$\|G_n(t)\| \leq [(\Lambda(\rho_5) + \Delta(\rho_5)t_0)\aleph_5]^{n+1}\theta_5 \tag{37}$$

$$\|H_n(t)\| \leq [(\Lambda(\rho_6) + \Delta(\rho_6)t_0)\aleph_6]^{n+1}\theta_6 \tag{38}$$

$$\|U_n(t)\| \leq [(\Lambda(\rho_7) + \Delta(\rho_7)t_0)\aleph_7]^{n+1}\theta_7 \tag{39}$$

$$\|W_n(t)\| \leq [(\Lambda(\rho_8) + \Delta(\rho_8)t_0)\aleph_8]^{n+1}\theta_8 \tag{40}$$

Similarly, taking the limit on Eqs. $(34) - (40)$ as $n \to \infty$ and then using condition (29), we have $\|C_n(t)\| \to 0, \|E_n(t)\| \to 0, \|F_n(t)\| \to 0, \|G_n(t)\| \to 0, \|H_n(t)\| \to 0, \|U_n(t)\| \to 0$ and $\|W_n(t)\| \to 0$. Therefore, the existence of the system of solutions of system (3)– (4) is proved.

We now give conditions for the system of solutions to be unique.

Theorem 6. System (3) along with the initial conditions (4) has a unique system of solutions if the following conditions hold: $1 - \Lambda(\rho_i)\aleph_i + \Delta(\rho_i)\aleph_i t > 0, \quad for\ i = 1,2,\ldots,8, \quad).$ (41)

Proof. Assume that$\{\{S(t), E(t),\ I_1(t),\ I_2(t), T(t), R(t),\ S_r(t)\ ,\ I_r(t)\ \}$is another set of solutions of model (3)– (4) in addition to the solution set $\{SS(t), E(t),\ I_1(t),\ I_2(t), T(t), R(t),\ S_r(t)\ ,\ I_r(t)\ \}$ proved to exist in Theorems 4 and 5 then

$$S(t) - S_1(t) = \Lambda(\rho_1)(K_1(t,S) - K_1(t,S_1) + \Delta(\rho_1)\int_0^t (K_1(y,S) - K_1(y,S_1))dy, \tag{42}$$

Taking the norm and triangle inequality on both sides of Eq. (38), we have

$$\|S(t) - S_1(t)\| \leq \Lambda(\rho_1)\|K_1(t,S) - K_1(t,S_1)\| + \Delta(\rho_1)\int_0^t \|(K_1(y,S) - K_1(y,S_1)\| \quad (43)$$

Using the Lipschitz condition for the kernel K_1, we find

$$\|S(t) - S_1(t)\| \leq \Lambda(\rho_1)\aleph_1\|S(t) - S_1(t)\| + \Delta(\rho_1)\aleph_1 t\|S(t) - S_1(t)\| \quad (44)$$

If Eq. (40) rearranging we obtain

$$\|S(t) - S_1(t)\|1 - \Lambda(\rho_i)\aleph_i + \Delta(\rho_i)\aleph_i t \leq 0 \quad (45)$$

Finally, applying condition (41) for $i = 1$ to Eq. (45), we obtain

$$\|S(t) - S_1(t)\| = 0 \quad (46)$$

Hence $S(t) = S_1(t)$.

Applying a similar procedure to each of the following pairs

$(E(t), E_1(t)), (I_1(t), I_{1_1}(t))$, $(I_2(t), I_{2_1}(t)), (T(t), T_1(t))$ $(R(t), R_1(t)), (S_r(t), S_{r_1}(t))$

and $(I_r(t), I_{r_1}(t))$.

with inequality (41) $for\ i = 1,2,\ldots,8$, respectively, we have

$$E(t) = E_1(t), I_1(t) = I_{1_1}(t), I_2(t) = I_{2_1}(t), T(t) = T_1(t), R(t) = R_1(t),$$

$$S_r(t) = S_{r_1}(t)\ and\ \ I_r(t) = I_{r_1}(t). \quad (47)$$

Thus, the uniqueness of the system of solutions of the fractional order system is proved.

5 Data fitting and sensitivity analysis and parameter estimation

To estimate the remaining parameter values of our Lassa fever model (1), we applied the model to the cumulative number of reported cases from 2019 to 2021, provided by the Nigeria Centre for Disease Control. We developed a MATLAB program using ODE45 solvers and employed model data fitting techniques through conventional nonlinear least squares methods, as shown in Table 3. Figure 2 provides a pictorial representation of the data fitting for the model using the cumulative confirmed cases.

Using the parameter values obtained from data fitting, we conducted sensitivity analyses to assess the impact of different parameters on Lassa fever dynamics within the population. This approach enabled us to simulate various scenarios, providing insights into the potential outcomes and effectiveness of different intervention strategies.

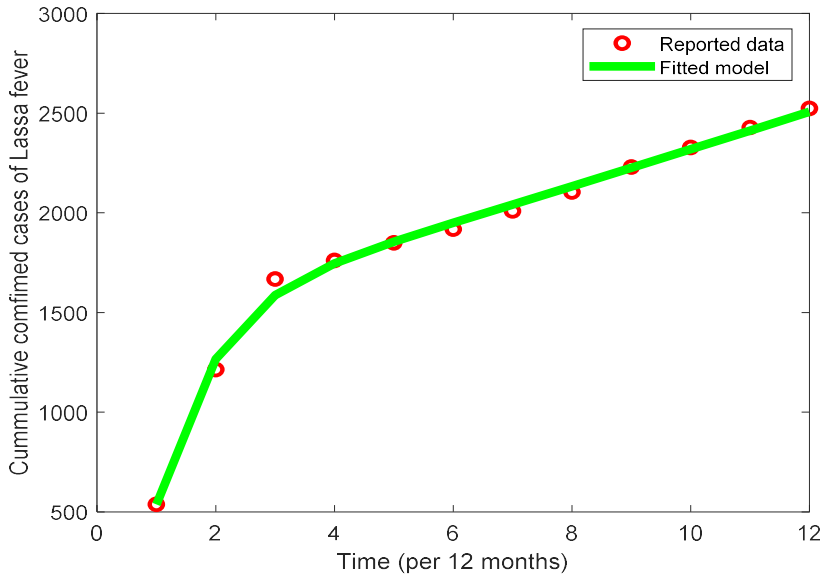

Figure 2. Data fitting of the Lassa fever model (1) using cumulative confirmed cases in

Parameter	Descriptions	Value	Source
Π	Rate of recruitment of susceptible human	68088	[37]
Λ	Rate of recruitment of susceptible mastomys rat's	557	[37]
μ	Natural death rate of human	0.000053	[37]
ψ	Natural death rate of mastomyns rat,	0.003	[36]
ϕ	Rate at which immunity wanes after recovery	0.7246	fitted
γ_1	Recovery rate of infected human population	0.7194	fitted
γ_2	Recovery rate of infected treated human population	0.7285	fitted
γ_3	Recovery rate of infectious deceased human population	1.5472	fitted
α_1	Disease induced death rates of individuals in $I_1(t)$	0.484	[36]
α_2	Disease induced death rates of individuals in $I_2(t)$	0.484	[36]
δ	Disease induced death rate of infected individuals in $T(t)$ compartment	0.8	[36]
ς	Rates at which infected deceased move to treatment class,	0.4325	fitted
ϱ	Rates at which infected humans move to treatment class	0.7774	fitted
ξ	Proportion of new exposed individual that become symptomatically infected	0.4182	fitted
ϑ	Rate at which an exposed individual becomes infectious	1.7169	fitted
φ	Transmission rates due to contact with infected deceased	0.4955	fitted
β_1	Rate of human-to-human contact	0.1682	fitted
β_2	Rate of Mastomys rat-to-human contact	0.0071	fitted
β_3	Rate of Mastomys rat-to-mastomys rat contact	0.0209	fitted

Table 3. Parameter values for the Lassa fever model (1)

6 Sensitivity analysis

Mathematical modeling of infectious diseases aims to understand how diseases spread in a population (Panovska-Griffiths, 2020). Sensitivity analysis is a key technique used to gain insights into disease dynamics by evaluating the impact of different parameters on the model. In this study, we used Partial Rank Correlation Coefficients (PRCC) within MATLAB R2022b to identify the most sensitive epidemiological parameters for controlling the Lassa fever (LF) outbreak (Musa et al., 2020).

Our analysis focused on how variations in each parameter affect the reproduction number, which helps in developing intervention strategies to control the spread of the disease. The sensitivity indices and parameter values are illustrated in Figure 3. The analysis revealed that positive values for parameters $\varphi, \vartheta, \xi, \beta_1$, β_3 and Λ were associated with an increased spread of Lassa fever. Conversely, a decrease in the negative values of parameters $\Pi, \gamma_1, \gamma_1, \gamma_1$, ς and ϱ was linked to a rise in transmission rates. The sensitivity indices showed that the natural mortality rate of rats ψ and the rodent-to-rodent transmission rate β_3 had the most significant effects. An increase in $\beta_3$results in a higher reproduction number, while an increase in ψ leads to a reduction in the rat population.

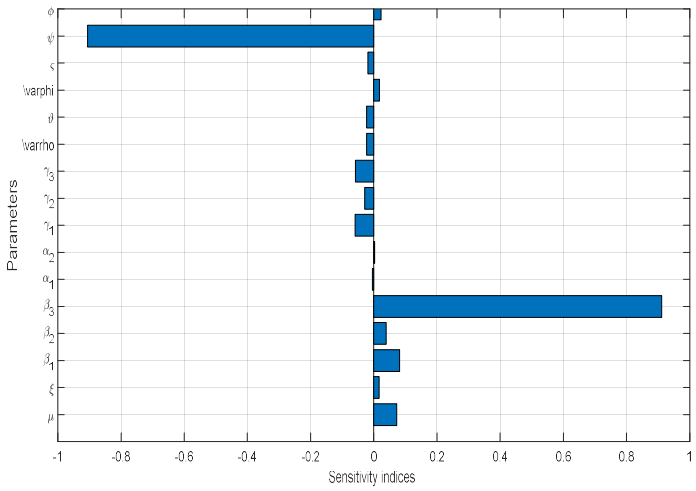

Figure 3. Sensitivity indices of the Lassa fever reproduction number using partial rank correlation coefficient (PRCC)

In conclusion, effective control measures should focus on reducing transmission likelihood and the rate of rat population recruitment. Strategies to achieve this include avoiding contact with infected corpses (through safe burial practices), promoting good hygiene, conducting educational campaigns, and using rodent traps or pesticides. These measures can significantly reduce the spread of Lassa fever among people.

7 Numerical computation

We now present the numerical results and simulations of the extended fractional order mathematical model in Caputo sense with the help of the derived algorithm and numerical coded written in MATLAB environment using the model equations and the values of the parameters in table 3.

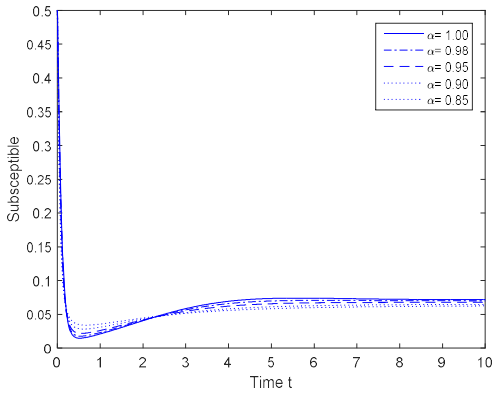

Fig. 1: Numerical Simulation of S(t) at different values of alpha values of alpha

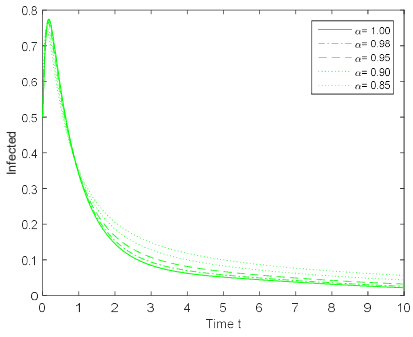

Fig. 2: Numerical Simulation of I(t) at different

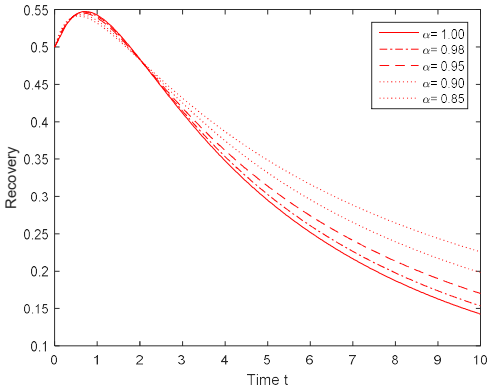

Fig. 3: Numerical Simulation of R(t) at different values of alpha

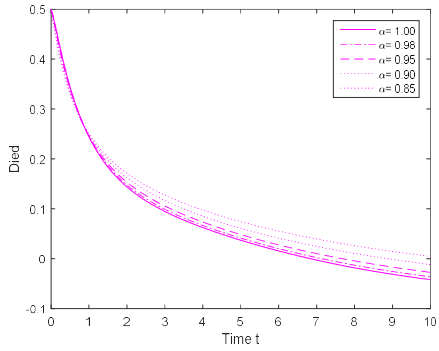

Fig. 4: Numerical Simulation of D(t) at different values of alpha

8 Result and discussion

In this paper, the dynamics of Lassa fever virus model are examined via Caputo–Fabrizio fractional order differential equation model approaches: Varying the values of fractional-order α for the FODE, Due to the lack of any disease control measures, the number of susceptible and infected population dramatically increases (see Figure 1), since both the susceptible and infected population live together in the environment that serves as a breeding ground for the bacteria and actively interact among themselves, Moreover, we can easily observe from Figure 6 and 7 that when $\alpha \rightarrow 1$ the Caputo–Fabrizio non-integer order derivative reveals more absorbing characteristics. Consequently, this causes the Lassa fever virus dynamic to stay at almost a constant rate for a long period of time (see Figures 2 and 4). However, the effect of increasing or decreasing infectious contact with environment in model (1) at different values of alpha in figure 3 indicate the vulnerability of all state variables.

9 Conclusion

In this paper, we examined the interaction between human and rodent hosts by formulating a non-linear deterministic mathematical model to describe the transmission dynamics of

Lassa fever, using demographic data from Nigeria. This model was extended to an integer type using the Caputo–Fabrizio fractional differential equation and analyzed using fixed point theory and an iterative method. The fractional model employs non-singular, exponentially decreasing kernels from the Caputo–Fabrizio fractional derivative, and we established the existence and uniqueness of solutions for the system. We identified the equilibrium points of the model and determined the conditions for local asymptotic stability of the disease-free equilibrium point. Numerical solutions of the fractional system were obtained and compared for different values of the fractional order, exploring the use of the Caputo–Fabrizio fractional derivative in modeling real-life problems involving memory effects.

The model was parameterized using cumulative confirmed cases of Lassa fever in Nigeria from January 2019 to December 2021, obtained from the Nigeria Centre for Disease Control database. Sensitivity analysis was conducted to evaluate the significance of each parameter on the transmission dynamics of Lassa fever in Nigeria. The analysis identified the transmission coefficients $\Lambda, \varrho, \varsigma, \varphi\ \beta_1, \beta_2\ and\ \beta_3$ as critical control parameters influencing the transmission of Lassa fever. The basic reproduction number, R_0, was found to be greater than one ($R_0 > 1$) , suggesting that Lassa fever is likely to remain endemic in Nigeria unless effective control methods are implemented to reduce R_0 below unity.

Our study explored the impact of controlled parameters on the total infected human and deceased populations. The results indicate that combining all possible transmission control measures significantly reduces the burden of Lassa fever more quickly in the population. Early treatment of infected individuals, personal hygiene, precautions by health workers, proper burial practices, educational campaigns, and the use of pesticides and rodent traps are essential strategies for reducing the number of infected individuals and containing the spread of Lassa fever in Nigeria.

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