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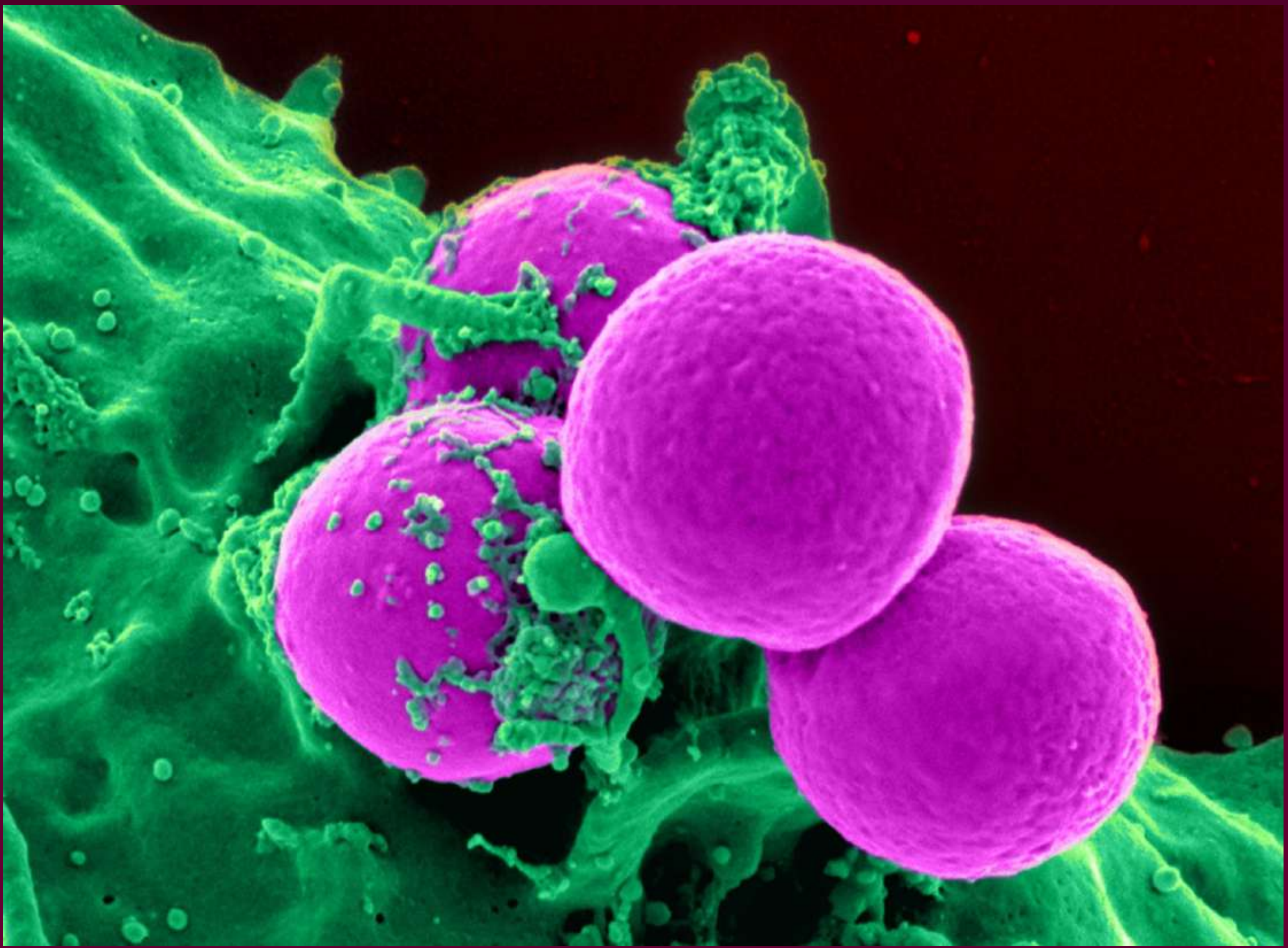

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Comparative Effect of Cooperative Learning Strategy and Lecture Method on the Academic Achievement of N.C.E Students in Electrolysis

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Abstract: *The study on the comparative effect of cooperative learning strategy and lecture method on the academic achievement of Nigeria Certificate in Education (NCE) 100 Level students in electrolysis; was a pretest posttest quasi-experimental-control group design. The population of the study comprised 243 students from physics and chemistry departments of Federal College of Education (Technical), Gombe, and from the population, 87 students were used as sample using purposive sampling technique. Electrolysis Achievement Test (EAT) was the instrument used for the study and data collection. Two research questions and two corresponding research hypotheses were raised. Descriptive statistic was used to answer the research questions while t-test was used to test the research hypothesis at 0.05 level of significance. The study showed significant difference in academic achievement between the two groups in favour of those taught using cooperative learning strategy and also revealed no significant difference on gender when taught using cooperative learning strategy. It was recommended that teachers should employ the use of cooperative learning strategy since it has been established to enhance students' academic achievement especially when they are allowed to interact freely with one another and in groups.*

Keywords: *Cooperative learning, Lecture Method, Academic Achievement, N.C.E. Students, Electrolysis*

Introduction

To overcome the challenges of the 21st century especially in the area of science and technology, there is the need to foster students centered approach to teaching and learning. Many teaching methods/strategies to teaching have been innovated by educators and stake holders alike, however, to select the best, appropriate and suitable method/strategy to teach a particular subject, topic or even concept has been a source of concern. Metha and Kulshrestha (2014) asserted that the teaching and learning process has become an issue of rational consideration and on critical query on all fronts, and there have been debates on the instructional materials and methods to provide to students at various levels world wide. There are growing researches within the scientific community and other areas on pedagogy that support new teaching approach with a view to ensuring quality of instructions. Effective teaching is based on several factors such as positive reinforcements, advance organizers, cues and feedback, higher questioning technique, positive classroom environment and cooperative learning (Najmonnisa, Amin and Saád 2016). Therefore, the 21st century teacher needs to be equiped with modern teaching strategies inorder to produce better results. According to Wolfensberger and Canella (2015), many teaching methods and strategies have been identified and practiced and have produced encouraging results, and among these is cooperative learning.

Najmonnisa et al (2016) define cooperative learning as a teaching method that helps students learn together in groups to maximise their learning with interest and motivation. Hence cooperative learning can be regarded as a method/strategy of teaching that provides the learners with the opportunity to learn together in pairs or small groups. http://courses.coe.asu,edu (n.d) identified various strategies that exist in cooperative learning, these are:

(i) Think-pair-share: - this is a method that allows students to engage in individual and small group thinking before they are asked to answer questions in front of the whole class.
(ii) Three-Step-Interview: - this is a strategy that is effective when students are solving problems that have no specific right answers.
(iii) Round table or rally table: - this is a type of cooperative learning structure that covers much content, builds team spirit and involves much writing.
(iv) Group investigations: - these are structured to emphasise higher order thinking skills such as analysis and evaluation. Here, students work to produce group project which they themselves choose to work on.
(v) Students Team Achievement Division (STAD):- this invoves students with varing academic abilities being assigned to 4 or 5 member teams to study what was initially taught by the teacher and to help each student attain his or her highiest level of academic achievement. Each member of the group earn certificate or recognition based on the degree to which all the team members over their past record.
(vi) Jigsaw II:- Here, each member is responsible for learning a specific topic, and after meeting with other group members who are expert in that area, this person return back to the group to present the topic or findings. All team members are required to also present their findings.
(vii) Round Robin Brainstorming or Rally Robin: - this is a strategy in which the class is divided into a group of 5 or 6 students per group with one group member as a recorder. Here, the question is asked by the teacher with many possible answers and the students are given time to think. After the think time, members share responses with one another and the recorder writes all the responses.

(viii) Three-Minute Review:- this is used when the teacher stop any time during lecture or discussion to allow students three minutes to review what was taught.

It is in this regard that the study on the comparative effect of cooperative learning and lecture method will be carried out among students in Colleges of Education students in Nigeria.

The lecture method in science is a method of teaching that emphasizes "talk - chalk" in the teaching of science subjects. More than 80% of scientific information, ideas, concepts, generalization and facts are verbally presented to students by the teacher (Abdullahi, 1982).The teacher does much of the activity in form of talking while the students are either passive or slightly involved. Olarenwaju (1994) sees lecture method as pure teacher centered approach where students are not given opportunity to ask questions or give feedback to the teacher. Here, the teacher talks and writes notes on the board while the students listen and copy down notes.The lecture method does not promote academic performance in science as observed by Abdullahi (1982).

Kuar (2011) in a study, described lecture as a model of teaching that is frequently critisized, but it is a fact that it has managed to survive so long in pace of technological developments and is still often used to teach organised bodies of knowledge which is an important part of the school curriculum at all levels, and they have continued as a primary form of instruction in colleges and universities; the same study also concluded that lecture method remain popular for several reasons which include among others:

(i) They are efficient, planning time is devoted to organizing the context and less attention is devoted to teaching strategy;
(ii) They are flexible and can be adapted to a wide range of subjects;
(iii) Most people can learn to lecture well enough to survive in a classroom as they are easier to learn than most instructional strategies;
(iv) They are easier for the teacher due to simply "telling" students about the subject and topic.

Therefore, lecture method can be considered as a popular teaching model in different fields of study such as in sciences, engineering, social sciences, medicine and arts in colleges and universities throughout the world.

Hence, this study compared the academic achievement of students taught electrolysis using the two discussed teaching methods.

Statement of the Problem

Nigeria is facing problems in the teaching of physics at both secondary and tertiary levels of education. It has been observed that the teaching of Physics suffers due to limited resources, equipment and latest physics books (Oladejo, Olusunde, Ojebisi and Ishola, 2011). According to Ogunleye and Babajide, (2011), Nigeria has witnessed persistent poor students' performance in Physics at the Senior School Certificate level. This has been linked to the adoption of instructional strategies which did not give enough consideration to learners' previous knowledge and how they reasoned in order for learners to construct their knowledge based on these. According to Adeyemi (2010), problems in teaching physics can be minimized by selecting suitable teaching methods.

Cooperative learning is one of those teaching strategies and is a pedagogical practice that has attracted much attention over the last decades because of a large body of research that indicates students gain both academically and socially when they have opportunities and when they interact with others to accomplished shared goals (Gilles and Boyle, 2009).

The study therefore taught an introductory physics course (electrolysis) at the Nigeria Certificate in Education (NCE) level and compared cooperative learning strategy and lecture method and its effects on the academic performance of Nigeria Certificate in Education (NCE) 100 Level students in Federal College of Education (Technical), Gombe.

Justification of the Study

In spite of the great emphasis on physics teaching and because of its central role in technological advancement, students are observed to do poorly in the subject. Many factors have been attributed to this ugly situation. These factors include poor teaching methods, lack of instructional materials, lack of functional laboratories, poor students' technological attitude and lack of qualified physics teachers. Many research studies have been carried out on other factors mentioned, however, the researcher has not been able to find similar studies among N.C.E students especially on the topic electrolysis" and hence this study intends to make its contribution in this area.

Objectives of the Study

The purpose of this study investigated the effects of cooperative learning and lecture teaching methods on the academic achievement of NCE I00 level students in physics and the specific objectives of the study were:

(i) To investigate the effects of cooperative learning and lecture teaching methods on the academic achievement of NCE I00 level students on the topic "electrolysis" in Fedreal College of Education (Technical), Gombe.

(ii) To find out the effect of cooperative learning on gender among NCE 100 level students on the topic "electrolysis" in Federal College of Education (Technical), Gombe.

Research Questions

The following research questions were made for the study:

i) Is there any significant difference in the mean score of NCE 100 level students taught electrolysis using cooperative learning and those taught the same topic using lecture method?

ii) What is the difference in the mean score of male and female NCE 100 level students taught electrolysis using cooperative learning teaching strategy in Federal College of Education (Technical), Gombe?

Null Hypotheses

The following null hypotheses were tested at $p \leq 0.05$ level of significance.

Ho_1: There is no significant difference in the academic achievement of NCE 100 level students taught electrolysis using cooperative learning and those taught the same topic using lecture method?

H_{02}: There is no significant difference in the academic achievement of NCE 100 students taught electrolysis using cooperative learning strategy on the basis of gender.

Scope of the Study

The study was delimited to only NCE I students studying Physics and Chemistry in Federal Colleges of Education (Technical), Gombe in North- Eastern Nigeria. The topic electrolysis selected for the study was based on the contents and objectives of the requirements in the curriculum of Nigerian Certificate in Education (NCE). The methods of instruction used in this study were cooperative learning and lecture methods of teaching.

Research design

The design for this study was a quasi-experimental-control groups design consisting of prettest and posttests. Pretest and posttest was administered to the experimental and control groups as recommended by Akuezuilo, (1993) and Musa (2000). An Electrolysis Achievement Test (EAT) was administered to both groups during the prettest in order to determine their knowledge of electrolysis, the groups equivalence and their ability before treatment and at the end of the treatment, posttest using (EAT) was again administered to the two groups to determine the significant difference if any in their mean academic performance in electrolysis.

Population and Sample of the Study

The study comprises all NCE students from Physics and Chemistry Departments of Federal College of Education (Technical), Gombe. There are 234 students in NCE I, II and III in both departments with 143 male and 91 female. The sampling technique was a purposive technique since the topic used for the study (electrolysis) is common to both groups at the NCE 100 level. The students sample was purposive too since the number of NCE 100 level students from both departments stands as 87, with 46 male and 41 female, hence an intact class was chosen for the study. Simple random samling technique was used to divide the students into the control and experimental groups so that 40 students were selected for the control while 47 were selected for the experimental group before the treatment.

Instrument, Validity and Reliability

The instrument used for data collection in this study was Electrolysis Achievement Test (EAT), it was constructed by the researchers and is comprised of forty multiple choice items with options A-E from the topic electrolysis as contained in the NCE minimum standard. The instrument was validated by senior lecturers in the department of physics and chemistry for clarity and relevevance. A prettest was conducted before treatment inorder to find out its reliability. A reliability coefficient of 0.88 was obtained using the split-half method which made the insrument reliable for the study.

Treatment and Administration of Instrument

Both the experimental and control groups were treated using cooperative learning and lecture method respectively for six (6) weeks. During this period, the topic electrolysis was taught by the researchers and each contact lasted for two hours. After the treatment both groups were posttested using EAT. During the treatment, the experimental group was divided into four (4) groups and the topics under electrolysis were assigned to each of the groups to deliberate in groups and were then presented as group presentation during each contact. Hence the group investigations strategy of cooperative learning was adopted in the treatment of the experimental group. This is structured to emphasise higher order thinking skills such as analysis and evaluation, which implies students work to produce group project which they themselves choose to work on under the guidance of the teacher. For this study, the group distribution is shown in table 1.

Table 1 Showing Students groups Distribution in Cooperative Learning

Group(s)	Topic(s) Distribution	Number students
A	Dynamics of charged particles (ions)	12
B	Examples of Electrolysis	12
C	Faraday's laws of Electrolysis and quantitative examples	11
D	Applications of Electrolysis	12

Data Analysis

The students' scores from the posttests served as the source of data for the study. Descriptive statistic was used in anwsering the research questions while t-test was used to test the research hypotheses at $P \leq 0.05$ served as the tools for the data analysis.

Analysis and Results Presentation

The analysis and presentation of the results were conducted using descriptive statistics in responding to the research questions and t-test was used to answer the research hypotheses.

Research question one

i) Is there any significant difference in the mean score of NCE 100 level students taught electrolysis using cooperative learning and those taught the same topic using lecture method?

To answer research question one, the mean score between the two groups was determined using descriptive statistics and the result is presented in Table 2

Table 2 Descriptive Statistics Results of Difference in Academic Achievements Between Students in the Control and Experimental groups

Group	N	Mean	Std Dev.	Mean Score Difference
control group	40	14.125	4.26	
				8.94
Experimental group	47	23.064	5.32	

The results in Table 2 showed that the control group has a mean of 14.125 while the experimental group has a mean of 23.064; the difference in mean was observed to be 8.94 and to test whether the difference is significant or not, null hypothesis one was formulated and tested using independent t-test statistic, the hypothesis reads:

Ho_1: There is no significant difference in the academic achievement of NCE 100 level students taught electrolysis using cooperative learning and those taught the same topic using lecture method?

Table 3: Comparison of the Academic Achievement between the Students in the Control and Experimental groups

Group	N	Mean	Std Dev	S.E	Df	t_{cal}	t_{crit}	Decision
Control Group	40	14.125	4.26	1.045				
					85	8.55	1.67	Rejected
Experimental Group	47	23.064	5.318	1.027				

$p \leq 0.05$

The results shown in table 3 revealed that t-calculated has a value of 8.55, while t-critical has a value of 1.67 for df = 85 at $P \leq 0.05$ and since t_{crit} is less than t-calculated, the null hypothesis is therefore rejected, this signifies that there is significant difference in the academic achievement of students taught electrolysis using lecture method and those taught the same topic using cooperative learning strategy.

Research question two

ii. What is the difference in the mean score of male and female NCE I00 level students taught electrolysis using cooperative learning teaching strategy in Federal College of Education (Technical), Gombe?

Table 4: Descriptive Statistics Results of Difference in Academic Achievements Between male and female Students in the Experimental group

Group	N	Mean	Std Dev.	Mean Score Difference
Male	22	24.091	5.23	
				1.93
Female	25	22.160	5.34	

The results in Table 4 showed that male group has a mean of 24.091 while the female group has a mean of 22.160; the difference in mean was observed to be 1.93 and to test whether the difference is significant or not, null hypothesis two was formulated and tested using independent t-test statistic, the hypothesis reads:

$\mathbf{H_{O2}}$There is no significant difference in the academic achievement of NCE I00 level students taught electrolysis using cooperative learning strategy on the basis of gender.

Table 5: Comparison of the Academic Achievement between the Male and Female Students in the Experimental group

Group	N	Mean	Std Dev	S.E	Df	t_{cal}	t_{crit}	Decision
Male Group	22	24.091	5.23	1.114				
					45	1.25	1.67	Accepted
Female Group	25	22.160	5.34	1.067				

$p \leq 0.05$

The results shown in table 5 revealed that t-calculated has a value of 1.25, while t-critical has a value of 1.67 for df = 45 at $P \leq 0.05$ and since t_{crit} is greater than t-calculated, the null hypothesis is therefore accepted, this signifies that there is no significant difference in the academic achievement of male and female students taught electrolysis using cooperative learning strategy.

Discussion of Results

The results in Table 3 indicated that there is significant difference in the academic achievement of those taught electrolysis using lecture method and those taught the same topic using cooperative learning strategy of teaching. This finding is in agreement with the results obtained by Ikechuwu, (2011), which maintained that cooperative learning enhances students' academic achievement and interest in chemistry and that it has no effect on gender when it compared with peer-teaching. In the same vain, Njoroge and Githua (2013) in a study on the effect of coopperative learning in mathematics on gender revealed that there was no statistical difference in academic achievement in mathematics. Jimoh, Idris and Olatunji (2016) also in a study on the effetcs of Jigsaw cooperative learning strategy and gender on students academic achievement in cost accounting in Colleges of Education in Ogun state revealed that students taught cost accounting jigsaw cooperative learning strategy performed better than those taught using lecture method and that cooperative has no effect on gender.

Summary and conclusion

The study was an investigation into the comparative effects of lecture method and cooperative learning strategy of teaching on the academic achievement on NCE I00 level students in electrolysis. The study was a pretest-posttest experimental-control group design. The population of the study comprised of all NCE I00 level students from physics and chemistry departments of Federal College of Education (Tech) Gombe. 87 students from the two departments were purposively used as sample for the study. Electrolysis Achievement Test (EAT) was the instrument used for the collection of data which was constructed by the researchers. It consists of forty (40) multiple choice questions based on the topic electrolysis with a reliability coefficient of 0.88. Descriptive statistic was used to answer the research questions while t-statistic was used to test the two hypotheses raised at 0.05 level of significance. It was concluded that there is significant difference in the academic achievement of those taught electrolysis using lecture method and those taught the same topic using cooperative learning strategy of teaching in favour of those taught using cooperative learning strategy. In the same vain, the study revealed that there is no significant difference in the academic achievement of male and female students taught electrolysis using cooperative learning strategy.

Recommendations

The study recommended that

(1) In large classess, students should be grouped into smaller groups so that they can engage themselves in discussions and later present their discussions in class.
(2) Teachers should employ the use of cooperative learning strategy since it has been established to enhance students' academic achievement especially when they are allowed to interact freely with one another and in groups.
(3) Students should also be allowed to make their own notes during self study.
(4) Teachers/Lecturers when teaching electrolysis should divide the topics into sections and distribute according to the sub-groupings.

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Application of Residue Calculus on Second Order Linear Homogeneous Differential Equation

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Abstract: *The focus of this study is to show the application of residue calculus of second order linear Homogeneous differential equations. Cauchy method was introduced to solve second order linear homogeneous differential equation. In comparing the method of finding the Cauchy's method for solving ordinary second order homogeneous differential and the order method both result are the same. The Cauchy's residue method is more direct, precisely, efficient, and time-saving. The result obtained shows that it has no complex solution. However both result are real and have their applications in electrical and mechanical engineering systems. Electrical application can be used in an electrical circuit such as Resistor, inductor and capacitor and the Mechanical can be used to describe acceleration, velocity and displacement. In short, the result has application in the field of engineering particularly Electrical and mechanical engineering.*

Keywords: *Second order differential. Residue calculus, Homogeneous Differential Equation*

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1. INTRODUCTION

In complex analysis, a field in mathematics, the residue theorem, sometimes called Cauchy's residue theorem(one of many things named after Augustin-Louis Cauchy), is a powerful tool to evaluate line integral of analytic function over closed curves; it can be used to compute real integral as well. It generalizes the Cauchy integral theorem and Cauchy's integral formula. From a geometrical perspective, it is a special case of the generalised stokes theorem. The analytical evaluation of a general contour integral with integral f(z) depends for its success on what are called the residue at the poles of f(z).The residue of a function f(z) at a pole of defined in terms of a special series expansion of f(z) about the pole called a Laurent series. The Laurent series represents an extension of the conventional Taylor series that is no longer applicable expansion of f (z) is required about a singular point. Various ways of obtaining Laurent series are described, and it is shown how a contour integral is related to the residue of the integrant f (z) that lie either inside or on the contour of integration different types of contour integral are evaluated and integration around a branch point of f(z) is considered. The mathematical field of complex analysis, contour integration is a method of evaluating certain integrals along path in the complex plane. Contour integration is closely related to calculus of residues a methodology of complex analysis. One use for contour integrals is the evaluation of integral along the real line that is not readily found by the using only real variable methods.

This work shows the application of residue calculus on second order homogeneous differential equation. Residue calculus is one of the most important notions of mathematics which find it application in all fields, especially in science and technology. The objective of this paper is to introduce Cauchy method for solving linear second order homogeneous differential equations with constant coefficients and its applications.

2. LITERATURE

An equation of the form $x''+Px'+Qx=0$ where P and Q are function of x, is said to be homogeneous differential equation, if it is not equal to zero is said to be non-homogeneous differential equation. Second order differential equations are classical methods which have wide area of applications in the field of science and technology. Especially in Engineering discipline. Generally, the differential equation may be ordinary or partial differential equation. [5]

Proposed was used On-chip Tunable second order differential equation solver Base on a silicon Photonic Mode-split micro-resonator with Tunable coefficients and system demonstration using the fabricated device is carried out for 10-Gb/s Gaussian and supper Gaussian in put pulse. The experimental results are in good agreement with theoretical prediction of the solution. [9]

Physics laws are generally written in the form of differential equations, science and technology are use differential equation the main concerned of differential equations are one the most important part of language of science and technology. [6]

Application of differential transform method for solving differential and integral equations was used to apply in area of Engineering and science using differential transform method was employed to solve Volterra integral equation of second kind. Taylor series polynomial or expansion was also used to construct analytical approximate solution of initial value problem. Differential transform method has been successfully use for finding solution of non-linear system of volterra integral equation and its powerful tools technique for obtained exact solution.[10]

Second order differential equation arises for the charges on a capacitor in unpowered RLC electrical circuit or freely oscillating frictional mass on a spring or for a damping pendulum.

Application of second differential equation was recently discuss [7], in the context, Schrödinger equation were discuss and condition under which the confluent, biconfluent and the generally, exact solution are transcendental functions and was a recursive ring and the Schrödinger equation where is extremely use full to have at one's disposal some. Application of the results to Schrödinger's equation was discussed and conditions under which the confluent, biconfluent, and the general Heun equation yield polynomial solution are explicitly given. [1]

Determining dynamic market Equilibrium price function was used to obtained the equilibrium price function over in dynamic market, by observing the price changes and changes in the level of rising prices. Second order non-homogeneous differential equation was used to determining Dynamic market Equilibrium price functions over a time by considering the price change and change in the level of rising price. [8]

Most of the cited literature they did not apply the method of Cauchy to solve second order ordinary differential equation, in other to solve the second order homogeneous differential equation. But they applied various methods to solve it.

3. PROCEDURE

GENERAL SOLUTION OF ORDINARY DIFFERENTIONAL EQAUTIONS

Here we introduce Cauchy's method for solving ordinary differential equations using residue. Specially, we will find the general solution for linear homogeneous differential equations with constant coefficients

$$a_0 y^n + a_1 y^{n-1} + \cdots + a_{n-1} y' + a_n y = 0 \qquad \textbf{(1)}$$

Where $a_0 = 1 \; and \; a_j\text{'s}, \; j = 1,2,3,\cdots$ are given constant s.

Theorem 1

Consider the differential equations with constant coefficients.

$$y^n + a_1 y^{n-1} + \cdots + a_{n-1} y' + a_n y = 0 \qquad \textbf{(2)}$$

Let *f* be an arbitrary function of the complex variable *z*, whose zeros do not coincide with the zeros of the polynomial

$$g(z) = z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_{n-1} z + a_n \qquad \textbf{(3)}$$

Then the general solution of (4.2.2) is given by

$$y(x) = \sum res\left[\frac{f(x)e^{zx}}{g(z)}\right] \qquad \textbf{(4)}$$

We now show that (4) is a solution of the homogeneous differential equation (2). We assume that

$$y = \sum res\left[\frac{f(x)e^{zx}}{g(z)}\right]$$

Then

$$y' = \sum res\left[\frac{f(x)e^{zx}}{g(z)}.z\right]$$

$$y'' = \sum res\left[\frac{f(x)e^{zx}}{g(z)}.z^2\right]$$

$$\vdots$$

$$y^k = \sum res\left[\frac{f(x)e^{zx}}{g(z)}.z^k\right] \quad (k \ =1, 2,3\ldots n)$$

Hence

$$y^n + a_1 y^{n-1} + \cdots + a_{n-1}y' + a_n y \qquad (\mathbf{5})$$

$$= \sum \operatorname{Re}s\left[\frac{f(z)e^{zx}}{g(z)}z^n\right] + a_1 \sum \operatorname{Re}s\left[\frac{f(z)e^{zx}}{g(z)}z^{n-1}\right] + \cdots + a_{n-1}\sum \operatorname{Re}s\left[\frac{f(z)e^{zx}}{g(z)}z\right] + a_n \sum \operatorname{Re}s\left[\frac{f(z)e^{zx}}{g(z)}\right]$$

$$= \sum \operatorname{Re}s\left[\frac{f(z)}{g(z)}e^{zx}\left(z^n + a_1 z^{n-1} + \cdots + a_{n-1}z + a_n\right)\right] = \sum \operatorname{Re}s\left[\frac{f(z)}{g(z)}e^{zx}g(z)\right]$$

$$= \sum \operatorname{Re}s\left[f(z)e^{zx}\right] = 0$$

Since f(z) is analytic. Thus, (4) is indeed a solution of (2), i.e. (4) is a general solution

4. RESULT

Problem 3.1: - The case of distinct real roots. We want to find the general solution of the differential equations

$$y'' - 7y' + 12 = 0$$

Let f(z) be any arbitrary entire function whose zero are 3 and 4

Let $g(z) = z^2 - 7z + 12$. Then $g(z) = (z-3)(z-4)$

Clearly the zeros of f (z) do not coincide with the zero of g(z). We know the general solution is given by

$$y = \sum \operatorname{Re} s\left[\frac{f(z)}{g(z)}\right]$$ By **(4)**

Where the summation is take over z = 3 and z = 4. So we have

$$y = \operatorname{Re} s\left(\frac{f(z)}{g(z)} e^{zx};3\right) + \operatorname{Re} s\left(\frac{f(z)}{g(z)} e^{zx};4\right)$$

$$= \lim_{z\to 3}(z-3)\frac{f(z)e^{zx}}{(z-3)(z-4)} + \lim_{z\to 4}(z-4)\frac{f(z)e^{zx}}{(z-3)(z-4)}$$

$$= \lim_{z\to 3}\frac{f(z)e^{zx}}{(z-4)} + \lim_{z\to 4}\frac{f(z)e^{zx}}{(z-3)} = -f(3)e^{3x} + f(4)e^{4x}$$

Let $let \quad c_1 = -f(3) \quad and \quad c_2 = f(4)$

Hence the general solution of differential equation is

$$y = c_1 e^{3x} + c_2 e^{4x}$$

Problem 3.2: - the case of repeated real roots. It is our objective to find the general solution of the differential equations.

$$y'' - 6y' + 9y = 0$$

Let f(z) be an arbitrary entire function whose zero's do not include 3, and let

$g(z) = z^2 - 6z + 9$ Then $g(z) = (z-3)^2$

The general solution is given by

$$y = \sum Res\left[\frac{f(z)}{g(z)} e^{zx}\right]$$

Then we have

$$y = \sum Res\left[\frac{f(z)}{(z-3)^2}e^{zx},3\right]$$

$$= \lim_{z\to 3}\frac{1}{(2-1)!}\frac{d}{dz}\left[(z-3)^2\frac{f(z)}{(z-3)^2}e^{zx}\right] = \lim_{z\to 3}\frac{d}{dz}\left[f(z)e^{zx}\right]$$

$$= \lim_{z\to 3}\left[f(z)xe^{zx} + f'(z)e^{zx}\right] = \left[f(3)xe^{3x} + f'(3)e^{3x}\right]$$

$let \quad f'(3) = c_1 \quad and \quad f(3) = c_2$

Thus, we have obtain the general solution,

$y = c_1 e^{3x} + c_2 xe^{3x}$

Problem 3.3

The equation $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc}i = 0$

Represent a current i flowing in an electrical circuit containing resistance R and inductance L and capacitance C connected in series. If R = 200 ohms, L = 0.20 henry and $C = 20\times10^{-6}$ *farads*, solve the equation for i given the boundary conditions that when t = 0, i = 0 and $\frac{di}{dt} = 100$

Solution

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{200}{0.20}\frac{di}{dt} + \frac{1}{0.20\times20\times10^{-6}}i = 0$$

$$\frac{d^2i}{dt^2} + 1000\frac{di}{dt} + 250000i = 0$$

$$g(z) = z^2 + 1000z + 250000 = 0$$

$$z = \frac{-1000 \pm \sqrt{1000000 - 1000000}}{2} = \frac{-100}{2} = -500$$

$z = -500$ Of order 2

Res (-500)

$$= \lim_{z \to -500} \frac{1}{(2-1)!} \frac{d}{dz}\left[\frac{(z+500)^2 f(z)e^{zt}}{(z+500)^2}\right] = \lim_{z \to -500} \frac{1}{(2-1)!} \frac{d}{dz}\left[f(z)e^{zt}\right]$$

$$= \lim_{z \to -500}\left[f(z)te^{zt} + f'(z)e^{zt}\right]$$

$$i = f(-500)te^{-500t} + f'(-500)e^{-500t}$$

$$i = c_1 te^{-500t} + c_2 e^{-500t}$$

When t =0 , i = 0 and c = 0

$$\frac{di}{dt} = -500c_1 te^{-500t} + c_1 e^{-500t} - 500c_2 e^{-500t}$$

$$100 = c_1 - 500c_2 \Rightarrow c_1 = 100$$

$$i = 100te^{-500t}$$

Problem 3.4

$L\frac{d^2 i}{dt^2} + R\frac{di}{dt}\frac{1}{C}i = 0$ is an equation representing current i in an electrical circuit. If inductance L is 0.25 henry, capacitances C is 29.76×10^{-6} farads and R is 250 ohms, solve the equation for i given the boundary condition that when t = 0, i = 0 and $\frac{di}{dt} = 34$

Solution

$$0.25\frac{d^2 i}{dt^2} + 250\frac{di}{dt} + \frac{1}{29.76 \times 10^{-6}} = 0$$

$$0.25\frac{d^2i}{dt^2}+250\frac{di}{dt}+3360.215054=0$$

$$f(z)=Lz^2+Rz+\frac{1}{C}$$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-R\pm\sqrt{R^2-\frac{4L}{C}}}{2a}=\frac{-250\pm\sqrt{250^2-\frac{4(0.25)}{29.76\times10^{-6}}}}{2(.25)}$$

$$z=\frac{-250\pm\sqrt{28897.84946}}{0.5}=\frac{-250\pm169.99}{0.5}=\frac{-250\pm170}{0.5}$$

$$z=\frac{-250+170}{0.5}\ or\ z=\frac{-250-170}{0.5}$$

$$z=\frac{-80}{0.5}\ or\ z=\frac{-420}{0.5}$$

$$z=-160 or\ z=-840$$

Res (-160) + Res (-840)

$$i=\operatorname{Re}s\left(\frac{f(z)}{g(z)}e^{zt},-160\right)+\operatorname{Re}s\left(\frac{f(z)}{g(z)}e^{zt},-840\right)$$

$$i=\lim_{z\to-160}\frac{(z+160)f(z)e^{zt}}{(z+160)(z+840)}+\lim_{z\to-160}\frac{(z+840)f(z)e^{zt}}{(z+840)(z+160)}$$

$$i = \frac{f(-160)e^{-160t}}{680} + \frac{f(z)e^{-840t}}{-680}$$

$$i = c_1e^{-160t} + c_2e^{-840t}$$

$$0 = c_1 + c_2 \ \Rightarrow c_2 = -c_1$$

$$\frac{di}{dt} = -160c_1e^{-160t} - 840c_2e^{-840t}$$

$$34 = -160c_1 - 840c_2$$

$$34 = 680c_1$$

$$c_1 = \frac{1}{20}$$

$$but \quad c_2 = -c_1$$

$$c_2 = \frac{-1}{20}$$

$$i = \frac{1}{20}\left(e^{-160t} - e^{-840t}\right)$$

Problem 3.5

The displacement s of a body in a damped mechanical system, with no external forces satisfies the following differential equation

$$2\frac{d^2s}{dt^2} + 6\frac{ds}{dt} + 4.5s = 0$$

Where t represent time, it initially condition, when t = 0 , s = 0 and $\frac{ds}{dt} = 4$ solve differential equation of S in terms of t.

Solution

$$2\frac{d^2s}{dt^2} + 6\frac{ds}{dt} + 4.5s = 0$$

$$g(z) = 2z^2 + 6z + 4.5$$

$$z = \frac{-6 \pm \sqrt{36-36}}{4} = \frac{-3}{4}$$

Res (-3) of order 2

$$s = \lim_{z \to -\frac{3}{2}} \frac{d}{dz}\left[\frac{(z+3)^2 f(z) e^{zt}}{(z+3)}\right] = \lim_{z \to -\frac{3}{2}} \frac{d}{dz}\left[f(z) e^{zt}\right]$$

$$s = \lim_{z \to \frac{-3}{2}} \left[f(z) t e^{zt} + f'(z) e^{zt}\right]$$

$$s = f\left(\frac{-3}{2}\right) t e^{\frac{-3}{2}t} + f'\left(-\frac{3}{2}\right) e^{\frac{-3}{2}t}$$

$$s = c_1 t e^{-\frac{3}{2}t} + c_2 e^{-\frac{3}{2}t}$$

When t = 0 and s =0 then $c_2 = 0$

$$\frac{ds}{dt} = -\frac{3}{2} c_1 t e^{-3t} + c_1 e^{-3t} - \frac{3}{2} c_2 e^{-3t}$$

$$4 = c_1 - \frac{3}{2} c_2$$

$$c_1 = 4$$

$$s = 4te^{-3t}$$

5. CONCLUSION AND RECOMMENDATION

In comparing the method of finding the Cauchy's method for solving ordinary second order homogeneous differential and the order method both result are the same. The Cauchy's residue method is more direct, precisely, efficient, and time-saving. The result obtained shows that it has no complex solution. However both result are real and have their applications in electrical and mechanical engineering systems. Electrical application can be used in an electrical circuit such as Resistor, inductor and capacitor and the Mechanical can be used to describe acceleration, velocity and displacement. In short, the result have application in the field of engineering particularly Electrical and mechanical engineering.

It has been recommended that the method should be applied in solving problems in engineering and other related fields.

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Physicochemical Analysis of Some Sachet Water Samples Available in Maiduguri, Borno State

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Abstract: *The physicochemical analysis of some sachet water samples available from four different manufacturing companies available in Maiduguri, Borno State. The sachet waters are Rampoly water, Kuru water, Rahama water and Fell Free water labeled as sample 1, 2, 3, and 4 respectively. The physicochemical parameters of the samples were determined using assorted standard methods in which turbidity was reported to be 0.00, 0.00, and 0.00NTU for samples 1, 2, and 3 respectively which are absent or not detected. Similarly, sample 4 had a low value of 0.26 NTU. Some heavy metals were also estimated and the mean values of the metals Fe and Zn for the study samples were obtained to be 1.02mg/l and 0.59mg/l respectively. Pb has not been detected in all the samples. Other parameters were relatively within the condonable ranges of the WHO and NAFDAC standards for permissibility of drinking water and domestic uses.*

Keywords*: Physicochemical, concentration, impurities, metals, mean value, heavy metals.*

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INTRODUCTION

Water is the basic needs for living organisms in the world. People consume water daily for domestic, industrial, and agricultural activities. The availability of good quality water is an indispensable feature for preventing diseases and improving quality of life (Oluduro and Adenye, 2007). Natural water contains some types of impurities whose nature and amount vary with source of water. Metals for example, are introduced into aquatic system through several ways, which include, weathering of rocks and leaching of soils, dissolution of aerosol particles from the atmosphere and from several human activities, including mining, processing and the use of metal-based materials (Ipinmoroti and Oshodi, 1993; Asaolu et al. 1997). Metals after entering the water, many can be taken up by fauna and flora and eventually, accumulate in marine organism that are consumed by human being (Asoalu et al, 1997).

The increased use of metal-based fertilizer in agriculture revolution could result in continued rise in concentration of metal pollutions in fresh water reservoir due to the water run-off. Also fecal pollution of drinking water causes water-borne disease which has led to the death of millions of people both in cities and villages (Asoalu, 2008). The discharge of these wastes may affect the aquatic of such river adversely and alter the chemical composition of the river (Adewoye, 1998). Water quality characteristic of aquatic environment arise from a multitude of physical, chemical and biological interaction (Deuzane, 2007). The water bodies, rivers, lakes, dams and estuaries are continuously subject to dynamic state of change with respect to the geological age and geochemical characteristics. This is demonstrated by continuous circulation, transformation and accumulation of energy and matter through the medium of living organisms and their activities. The dynamic balance is the aquatic ecosystem is upset by human activities resulting in pollution, which is manifested dramatically as fish kill, offensive taste, odour , colour and unchecked aquatic weeds (Asoalu, 2008).

SOURCE OF WATER

Basically, there are three main sources of water, these are:

- Rain-water
- Surface water
- Underground water

Rain water: Is regarded as the pure form of natural water, however, it can be contaminated by dissolved gases such as carbon dioxide (CO_2) and sulphur dioxide (SO_2) which combine with hydrogen ion (H^+) to form acid rain (Holderness, 2012).

Surface water: Surface water is mainly water from rivers, lakes, streams; oceans etc. surface water is the water, which falls to penetrate into the soil, but stays stagnant or flows along surface of the ground for some time (Howard, 2015).

Underground water: Underground water is found naturally as springs and artificially as well or borehole water. Underground water acts as a reservoir since its availability is not seasonal as rain or surface water (Howard, 2015).

MATERIALS AND METHODS

SAMPLE COLLECTION

This piece of work deals with analysis of some sachet waters available in Maiduguri town of Borno state. In view of this, four samples were bought from different places and of different sources. These are Rampoly water, Kuru water, Rahama water and Feel Free water. These samples were obtained in Maiduguri town. In other to avoid contamination a sterilized and labeled containers 1, 2, 3 and 4 were used for identification and were transported to NAFDAC Maiduguri north east zonal office, Geology Department in the University of Maiduguri and Maiduguri Water Treatment Plant for further analysis and processing.

SAMPLE ANALYSIS

A total of seven physical parameters were determined in water samples using HANNA products, for temperature, pH, Dissolved Oxygen (DO), Total Dissolved Solid (TDS), conductivity and turbidity. Three heavy metals were determined in the same samples (Zn, Fe and Pb), using LaMotte smart spectrophotometer.

PROCEDURES

The following procedures were applied in the determinations of the heavy metals in water samples.

DETERMINATION OF LEAD

A universal sample tube was rinsed with 10ml of sample water and scan blanked then it was removed 5ml of sample was removed using a syringe and was discarded, 5ml of the sample in the tube was transferred into the tube. A 5ml buffered ammonium chloride was added to fill the 10ml line of the tube; the sample was swirl to mix. 3 drops of 10% sodium cyanide was added. Sample was swirled to mix again. A 0.5ml pipette was used to add 0.5ml PAR indicator then swirled again. Another 0.5ml pipette was used to add 0.5ml stabilizing agent. The sample was capped and mixed then inserted into the sample chamber and scanned and the result was recorded as reading A. sample was removed from the sample chamber, three drops of DDC reagent was added. The sample was capped and swirled then subjected for analysis. The result was recorded as reading B.

mg/l lead = reading A – reading B.

DETERMINATION OF ZINC

A universal sample tube was rinsed with sample water and filled with it to the 10ml line. It was inserted into the sample chamber and scan blanked at the zinc test menu. The sample was removed from the chamber and a 0.5g sodium ascorbate powder was added to the sample using a 0.5g spoon. The sample was capped and shook vigorously then 3 drops of 10% sodium cyanide was added. The sample tube was capped and swirled. 1ml of dilute zinc indicator solution was also added using a 1ml pipette the sample tube was capped and mixed again. A plain pipette was used to add four drops of 37% formaldehyde solution, the sample was capped and inverted 15 times then it was inserted into the sample chamber and scanned, the result was recorded in mg/l.

DETERMINATION OF IRON

A sample tube was rinsed using the sample water then it was scanned blank at the iron test menu. Sample was removed and 0.5 ml of iron reagent was added to the sample using a 0.5ml pipette. The sample was capped and mixed; a 0.1g iron 2 powder was added. Sample was capped and swirled then shook vigorously for three minutes for maximum colour development sample was immediately inserted into the sample chamber and scanned. The result was recorded in mg/l.

DETERMINATION OF ELECTRICAL CONDUCTIVITY pH, TDS AND TEMPERATURE

The sample cell was filled to 25ml mark, the switch of the combo pH and EC was turned on. The sensitive probe was inserted into the sample water, using the set/hold bottom scroll and gets the parameters, the instrument then displays the values of the parameters.

DETERMINATION OF TURBIDITY (USING TURBIDITY METER)

A sample cell was rinsed with sample water and 10ml of the sample water was put and covered. It was then slot into the chamber and the power was turned ON, it displays SIP (sampling in progress) for some minutes and later the reading was displayed.

DETERMINATION OF DISSOLVED OXYGEN (DO)

Using an instrument Mettler Toledo, a beaker was filled with sample water and the instrument was inserted into the water and was turned ON. The reading is then displayed on the meter.

DETERMINATION OF BIOLOGICAL OXYGEN DEMAND (BOD)

The Biological Oxygen Demand was determined by measuring the Dissolved Oxygen (DO) level in the sample collected using an instrument mettler Toledo and comparing it to the dissolve oxygen level in a sample that was collected at the same time but incubating under specific condition (wrapped with black polyethene bag to prevent penetration of light for 5 days.

RESULTS AND DISCUSSION

TABLE 1 SHOWS THE PHYSIO-CHEMICAL PARAMETERS OF THE SAMPLES

Parameters	Sample 1	Sample 2	Sample 3	Sample 4	Standard value
Temperature (^oC)	28.9	30.3	29.9	28.1	Ambient
pH	7.01	7.21	7.81	7.43	7.0 – 8.5
TDS mg/l	7	81	37	85	100
Conductivity (NS/cm)	15	162	75	174	100 – 500
Turbidity (NTU)	0	0	0	0.26	5
DO (mg/l)	2.0	2.1	1.8	1.7	0.5 – 2
BOD (mg/l)	0.7	1.0	0.9	0.4	0.5 - 2

Table 2 shows the concentration of the heavy metals Fe, Pb and Zn in the four samples.

Metals	Sample 1	Sample 2	Sample 3	Sample 4	Standard Values
Fe (mg/l)	0.19	0.16	0.74	0.03	0.30
Pb (mg/l)	ND	ND	ND	ND	0.01
Zn (mg/l)	0.22	0.07	0.23	0.07	5.00

ND = Not Detected

DISCUSSION

The results as observed in table 1 presented fair criteria especially with respect to temperature, pH, conductivity and BOD which had all the value within the acceptable ranges for drinking water and aquatic lives. Sample 4 has the highest value for Total Dissolved Solid (TDS) and conductivity, although all fall below the WHO acceptable values of 100mg/l and 100-500mg/l for TDS and conductivity parameters respectively. Sample 2 gives the highest value of 2.1NTU for DO followed by sample 1 with 2.0NTU and least value was observed from sample 4 with 1.7NTU as in line with the standard value. As recommended by both WHO and NAFDAC (Federal Ministry of Environment, 1992).

Similarly, table 2 shows the metal concentrations of the studied samples with respect to Fe, Pb and Zn. Iron (Fe) was found to be lower of all the samples estimated with the value of 0.19mg/l, 0.16mg/l and 0.03mg/l with exception of sample 3 which is far above with the value 0.74mg/l.

Lead (Pb):- the concentration of Pb in all the samples were not detected, while the permissible value of WHO standard is 0.01mg/l, which is very negligible. This shows clear indication that all the samples are good for health, because lead (Pb) is very toxic and dangerous. Zinc (Zn): - The concentration of zinc in all the samples considered, though all the values fall below the admissible limit of 5.0mg/l, the highest value of 0.23mg/l was recorded in sample 3 and the least 0.01mg/l was seen in sample 2.

In conclusion, the concentration of the determined heavy metals Fe, Pb and Zn for the four samples under study with least concentration not health threatens except Fe in sample 3 which presented value above WHO standard.

CONCLUSION

From all the water samples analyzed, it shows that only Fe in sample 3 exceeds the WHO standard values, but for the rest of the samples they all falls within the acceptable ranges. This show a clear indication that the water samples are free from contamination and are healthy and safe for drinking.

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Determination and Health Implications of Heavy Metals on the Clothes of Mechanics in Buzaye Automobile Village, Sokoto

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Abstract: *The deposition of heavy metals on the clothes of local mechanics in Buzaye mechanic village of Sokoto was conducted using Atomic Absorption Spectroscopic method. Three samples of clothes were collected from fourteen workshops and samples were analyzed in triplicates. The assay revealed the present of heavy metals on the order of Zn>Ni>Cd>Pb. The same pattern was observed for average Zn concentration (µg/g) ranging from 16.5015 in sample K to 27.5777 in sample N. The average Zn concentration (µg/g) was in the order K (16.5015) > B (21.467) > L (23.3612) > I (24.4081) > D (25.8512) > M (26.1606) > A (26.3168). Other notable concentrations in an increasing order were samples H (26.9444) > C (26.9675) > F (27.1670) > E (27.4157) and lastly N (27.5777). The range and standard deviations of the metals is from 26.05±2.62 for Zn and 2.94±2.92. Analysis revealed a strong correlation of the metals from one automobile workshop to another. The study concluded that the interaction between the metals and the absorption ratio by body skin is highly probable.*
Key words: Workshops, Mechanics, Clothes, Health hazards and Heavy metals

1.0 Introduction

The dramatic increase in number of fairly used cars flying roads in most developing countries like Nigeria is alarming. These vehicles popularly known as Tokumbo cars in Nigeria pose serious environmental threat due to air pollution and in addition, automobile mechanic workshops had been established in recent years. During overhauling of these vehicle engines, metal fabrication and automobile panel beating, reasonable amount of spent engine oil and metals fillings are deposited on top soil while painting of vehicles and tyre vulcanizing are other activities that negatively affect the quality of soils around automobile workshops (Adewole and Uchegbu, 2010). Vehicle parts like break lining are known to contain copper which provides mechanical strength and assist in heat dissipation (Ukabiala *et al.,* 2009). Another author Cunningham *et al.,* (1975); reported that human activities such as metal smelting industries, coal combustion, auto emissions and application of commercial fertilizers, liming materials, sewage sludge, animal waste and irrigation water.

Like most of the developing cities in Nigeria, Sokoto is one of the are as were traffic density is increasing everyday. This is because of its closeness to the border region and a major entry route of these Tokumbo cars into the country. Buzaye is the biggest and the busiest automobile workshop in Sokoto metropolis and major mechanical workshop as a stop-over workshop. Table 1.0 presented the vehicular density of vehicles into the workshop. The workshop consisted of painters, mechanics, electric rewires, panel beaters, spare parts sellers and so on.

Table 1.0: Vehicles and Motorcycles tally entering the workshop in hours interval recorded during the sampling time of four hours (11:40 – 15:40 on 22nd Aug, 2017).

Sampling time	Motorcycle	Personal car	Bus and alike	Truck & van
0-60 mins	7	21	5	3
60-120 mins	9	28	1	6
120-180 mins	17	37	8	2
180-240 mins	5	26	3	0
Total	**38**	**112**	**17**	**8**

In most African countries, automobile mechanics are not using the needed safety kits such as eye goggles, uniform and so on. In fact, mechanics use the same cloth for a very long time without washing. In addition, these mechanics due to lack of equipment and working tools are using hands in most of their work resulting in deposition of different contaminants such as paints, PMS and used engine oils on their cloth and sweat as well. Hence, trace metal complex and dyes could be extracted from the fabrics by sweat solutions (Saracoglu *et al.,* 2003). In a complex mixture of different solvents as on the clothes of automobile mechanics there is a very high possibility of chemical leaching of heavy metals and their probable adsorption into the epidermal layer of the skin.

Occupational exposure to heavy metals has been widely reported by many authors (Nowak, 1994; Samanta *et al.,* 2004). Workers of the mining and production of cadmium, chromium, lead, mercury, gold and silver have been reported to be thus exposed; also inhabitants around industrial sites of heavy metal mining and processing, are exposed through air by suspended particulate matters (Heyer, 1985; Ogwuegbu and Muhanga, 2005). Most literature work on automobile workshops were based on the availability of heavy metals in soils of automobile workshops for example, the chemical fractionation of heavy metals in soils around the vicinity of automobile mechanic workshops in Kaduna metropolis, Nigeria was investigated by Achi *et al.*, (2011); and discovered that the highest percentage of metals were concentrated in the residual fractions.

To effectively predict the degree and magnitude of health risk it is important to studies the inherent hazards of a pollutant, exposure levels and population characteristics are very essential in drawing a final conclusion of the health risk of any pollutant. Studies from the field and laboratory experiments showed that accumulation of heavy metals in a tissue is mainly dependent on media concentration of in which metals are exposed and exposure period; although some other environmental factors such as pH and temperature play significant roles in metal accumulation (Jeffree *et al.,* 2006; Singh, R.K. *et al*., 2006).

The fear of this entry root of heavy metals into the body system of this group of automobile mechanics instigated this study to provide baseline information on the contamination of heavy metals (Zn, Ni, Pb and Cd) on the clothes of local automobile mechanics and to further highlight some health risk associated with this attitude.

2.0 Materials and methods

All the reagents used were analytical grade (AnalaR) chemicals and all the glassware, containers and tools were washed with liquid detergent first, rinsed with 20% (v/v) nitric acid and finally rinsed with distilled water. The containers and glassware were kept in oven until needed. Distilled water was used throughout the work (Tsafe *et al.,* 2012). Fourteen samples from different workshops were collected irrespective of the nature of work of the mechanics and for each of the samples it was divided into three parts and treated as a separate sample without any treatment.

2.1 Materials Digestion

1 g of each of the pieces of cloth from mechanics was digested with using nitric acid and hydrochloric acid in a ratio of 3:1 in digestion vessels until a clear solution was obtained. The digested clothes were filtered and made up to 100 ml with distilled water. Each sample was treated in triplicates and analyzed for heavy metals (Zn, Ni, Pb and Cd) using a Shimadzu AA 6800 Atomic Absorption Spectrophotometer at the National Research Institute for Chemical Technology-Zaria. Statistical analyses of the data was conducted and presented.

Fig. 1: Sample collected before digestion

3.0 Results and Discussion

The concentration of Zn (µg/g) was presented in table 2.0 showing, the average and the standard deviation of each sample. The range of the results for Zn (ppm) was from 16.38 in sample K to 27.87 in sample N. The same pattern was observed for average Zn concentration (µg/g) ranging from 16.5015 in sample K to 27.58 in sample N. The average Zn concentration (µg/g) was in the order K (16.50) > B (21.467) > L (23.36) > I (24.41) > D (25.85) > M (26.16) > A (26.32). Other notable concentrations in an increasing order were samples H (26.94) > C (26.97) > F (27.17) > E (27.412) and lastly N (27.58). Zinc, Cd, Pb, Cu are fuel additives and can easily be released into the atmosphere and carried to the soil through rain and wind (Mohammed *et al.,* 2005) as well can be deposited on automobile parts such as exhaust system even at much higher concentrations. Zn can be considered as one of the heavy metals which are components of tyre and engine and can be released during abrasion and wears (Odoh et al., 2011). The little variation in the results of Zn could be attributed to the period of exposure of the materials to contamination, since its expected that the longer the period of exposure, the higher the accumulation of trace metals not only in Zn.

Generally, toxic heavy metals such as Cd and Zn pose health hazards, if their concentrations exceeds the maximum permissible limits. However, the tendencies of accumulation over a long period of time is enormous. Akan (2009); discovered high concentrations of Zn in tissues of different categories of fishes with highest concentrations in the liver, gills, stomach, kidney and bone in descending order. Zinc oxide is the major cause of zinc shakes or Monday morning fever as is popularly called, a disease that is associated with acute occupational heavy metal exposure (Samara and Richard, 2006). The recommended Daily intake of Zinc is 150 ug/day (WHO, 1987), even though zinc deficiency facilitates effects of some toxic metals like lead.

Lead concentration on the clothes of local mechanics is presented in table 3.0. The concentration of lead was lowest in sample K (16.38 ppm) and highest was found in sample N (27.87 ppm), with an average concentration ranging from 16.50 ppm to 27.58 ppm. A comparative studies on nonoccupationally exposed residents showed higher in concentration in elements such as Pb, being 20 times higher and Cd being 10 times higher (Rodushkin and Axelsson, 2000) and thus the content of toxic metals suggested high levels of exposure to heavy metals (Elijah *et al.,* 2010). Lead toxic effects are vary in human from central nervous system in children, shortening the gravity, decrease in birth weight and, retardation of mental development when concentration is ≥ 500ug/l in blood ((Atanaskova, 2011; Mukesh *et al*., 2008). The effects of lead are manifested primarily in nervous haemopietic, urinary and genital systems and the kidney gets damaged due to decrease in blood hemoglobin (Ch.Subba 2010; Baykov *et al.,* 1996). In addition, lead to paints, dyes, and gasoline have created an epidemic of lead poisonings. Lead in its organic form is better absorbed through the skin (Samara and Richard, 2006). Toxicological effects of lead on man include inhibition of haemoglobin formation, sterility, hypertension and mental retardation in children.

Cadmium

The kidney is considered the critical target organ for the general population as well as for occupationally exposed populations resulting to renal failure while chronic obstructive airway disease is associated with long-term high-level occupational exposure by inhalation (FAO/WHO, 2011).

Cadmium, lead, copper and zinc are the major metal pollutants of the auto-mechanical workshop and are released from fuel burning, wear out of tyres, leakage of oils and corrosion of batteries and metallic parts such as radiators (Dolan *et al*., 2006). Refined foods are the major sources of Cd and the Daily dietary intake of Cd ranges from 40-50ug/day (WHO, 1987).

Other means of direct internal contamination include ingestion of food during routine work and dermal from dust that are likely to settle on sweat prone skin surfaces due to high temperature environment and heavy exercises (Arogunji 2007).

Sexuality, weight, age, (3), concentration, physicochemical properties, chemical bonds and their solution on absorption, accumulation, distribution in the body and physiological effects on metals (Dharib *et al*., 2003). Cd in chronic accumulation in the kidneys where it causes dysfunction if the concentration in the kidney cortex exceeds 200 mg/Kg fresh weight (Oyedele *et al.,* 2006) the amount of absorption and assembling depends on ecological, physical, chemical and biological condition and the kind of element and physiology of the body (Agbozu *et al.,* 2007). Chronic inhalation of cadmium causes both fibrotic and emphysematous lung damage, bone and Kidney resulting into renal failure. Itia-itia disease. (Samara and Richard, 2006).

Nickel

The concentration (µg/g) was presented in table 4.0 showing a range of nickel contents on the clothes of auto mechanics in buzaye was low. Nickel is used for nickel alloys, electroplating, machinery parts, stainless-steel, spark plugs and also as a catalyst while Nickel dermatitis, consisting of itching of the fingers, hands and forearms, it is the most common effect in human from chronic skin contact with nickel (Rajappa *et al.,* 2010). Nickel has been reported to be present in fuels as additives in low quantity and as trace metal in steel products. Heavy metals are toxicities are relatively uncommon but failure to recognize and treat heavy metal toxicities can result in significant morbidity and mortality (Samara and Richard, 2006). Genetics is an important factor because a higher incidence of heavy metal toxicity occurs in the African American population in the US because of delays in removing lead sources from lower socioeconomic areas (Samara and Richard, 2006).

4.0 Conclusions

The assay for the evaluation of heavy metals in pieces f mechanics clothes was conducted to acertain the level of possible exorsure to these metals. Metals including Zn, Ni, Cd and Pb were evaluated using atomic absorption spectroscopic technique. Results suggested the prevalence of these metals ranges from K (16.5015) to K (27.5777). Analysis revealed a strong correlation of the metals from one automobile workshop to another.

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Table 1: Total Zn Concentration (µg/g)

Sample No	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Sample A	26.4122	21.2414	26.7651	25.7644	27.3087	27.2336	27.1497	27.0022	24.3561	27.5459	16.5767	23.4335	26.2416	27.3695
Sample B	26.5395	21.7851	27.2393	25.9351	27.3377	27.1526	27.381	26.8692	24.6366	27.4129	16.3771	23.2774	26.0131	27.8727
Sample C	25.9987	21.3773	26.8981	25.8512	27.6037	27.1121	27.7744	26.9617	24.2288	27.4157	16.5478	23.3728	26.2243	27.4938
Average	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777
%RSD	1.0744	1.318	0.9072	0.33	0.5931	0.2277	1.1511	0.2531	0.8548	0.2767	0.6535	0.337	0.4862	0.9504
SD	0.009777	0.009784	0.008459	0.00295	0.005623	0.002139	0.01092	0.002358	0.007214	0.002627	0.003729	0.002722	0.004398	0.009063

Table 2: Total Ni Concentration (µg/g)

Sample No	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1st Sample	26.4122	21.2414	26.7651	25.7644	27.3087	27.2336	27.1497	27.0022	24.3561	27.5459	16.5767	23.4335	26.2416	27.3695
2nd Sample	26.5395	21.7851	27.2393	25.9351	27.3377	27.1526	27.381	26.8692	24.6366	27.4129	16.3771	23.2774	26.0131	27.8727
3rd Sample	25.9987	21.3773	26.8981	25.8512	27.6037	27.1121	27.7744	26.9617	24.2288	27.4157	16.5478	23.3728	26.2243	27.4938
Average	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777
%RSD	1.0744	1.318	0.9072	0.33	0.5931	0.2277	1.1511	0.2531	0.8548	0.2767	0.6535	0.337	0.4862	0.9504
SD	0.009777	0.009784	0.008459	0.00295	0.005623	0.002139	0.01092	0.002358	0.007214	0.002627	0.003729	0.002722	0.004398	0.009063

Table 3: Total Pb Concentration (µg/g)

Sample No	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1st Sample	26.4122	21.2414	26.7651	25.7644	27.3087	27.2336	27.1497	27.0022	24.3561	27.5459	16.5767	23.4335	26.2416	27.3695
2nd Sample	26.5395	21.7851	27.2393	25.9351	27.3377	27.1526	27.381	26.8692	24.6366	27.4129	16.3771	23.2774	26.0131	27.8727

3rd Sample	25.9987	21.3773	26.8981	25.8512	27.6037	27.1121	27.7744	26.9617	24.2288	27.4157	16.5478	23.3728	26.2243	27.4938
Average	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777
%RSD	1.0744	1.318	0.9072	0.33	0.5931	0.2277	1.1511	0.2531	0.8548	0.2767	0.6535	0.337	0.4862	0.9504
SD	0.009777	0.009784	0.008459	0.00295	0.005623	0.002139	0.01092	0.002358	0.007214	0.002627	0.003729	0.002722	0.004398	0.009063

able 4: Total Cd Concen ration (µg/g)

Sample No	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1st Sample	26.4122	21.2414	26.7651	25.7644	27.3087	27.2336	27.1497	27.0022	24.3561	27.5459	16.5767	23.4335	26.2416	27.3695
2nd Sample	26.5395	21.7851	27.2393	25.9351	27.3377	27.1526	27.381	26.8692	24.6366	27.4129	16.3771	23.2774	26.0131	27.8727
3rd Sample	25.9987	21.3773	26.8981	25.8512	27.6037	27.1121	27.7744	26.9617	24.2288	27.4157	16.5478	23.3728	26.2243	27.4938
Average	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777
%RSD	1.0744	1.318	0.9072	0.33	0.5931	0.2277	1.1511	0.2531	0.8548	0.2767	0.6535	0.337	0.4862	0.9504
SD	0.009777	0.009784	0.008459	0.00295	0.005623	0.002139	0.01092	0.002358	0.007214	0.002627	0.003729	0.002722	0.004398	0.009063

Table 4: Average Zn, Ni, Pb and Cd Concentrations (µg/g)

Sample No	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Average Zn (ppm)	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777
Average Ni (ppm) Average	26.5395	21.7851	27.2393	25.9351	27.3377	27.1526	27.381	26.8692	24.6366	27.4129	16.3771	23.2774	26.0131	27.8727
Pb (ppm) Average	25.9987	21.3773	26.8981	25.8512	27.6037	27.1121	27.7744	26.9617	24.2288	27.4157	16.5478	23.3728	26.2243	27.4938
Cd (ppm)	26.3168	21.467	26.9675	25.8512	27.4157	27.167	27.436	26.9444	24.4081	27.4591	16.5015	23.3612	26.1606	27.5777

Table 5: Two-tailed Correlations

		A	B	C	D	E	F	G	H	I	J	K	L	M	N
A	Pearson Correlation	1	.487	.458	.216	-.950	.589	-.824	-.434	.873	.275	-.578	-.347	-.631	.489
	Sig. (2-tailed)		.677	.697	.862	.201	.599	.383	.714	.325	.823	.608	.774	.565	.675
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
B	Pearson Correlation	.487	1	.999*	.958	-.191	-.419	.094	-.998*	.851	-.706	-.994	-.988	-.985	1.000**
	Sig. (2-tailed)	.677		.021	.185	.878	.725	.940	.038	.352	.501	.069	.098	.111	.002
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
C	Pearson Correlation	.458	.999*	1	.967	-.158	-.449	.126	-1.000*	.834	-.729	-.990	-.993	-.979	.999*
	Sig. (2-tailed)	.697	.021		.164	.899	.704	.919	.017	.373	.480	.090	.077	.132	.023
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
D	Pearson Correlation	.216	.958	.967	1	.099	-.662	.375	-.973	.665	-.880	-.921	-.991	-.894	.957
	Sig. (2-tailed)	.862	.185	.164		.937	.539	.755	.148	.537	.315	.254	.088	.296	.187
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
E	Pearson Correlation	-.950	-.191	-.158	.099	1	-.811	.959	.132	-.677	-.560	.295	.038	.358	-.194
	Sig. (2-tailed)	.201	.878	.899	.937		.398	.182	.916	.526	.622	.809	.976	.767	.876
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
F	Pearson Correlation	.589	-.419	-.449	-.662	-.811	1	-.943	.472	.120	.939	.319	.553	.255	-.416
	Sig. (2-tailed)	.599	.725	.704	.539	.398		.216	.687	.924	.224	.793	.627	.836	.727
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3

G	Pearson Correlation	-.824	.094	.126	.375	.959	-.943	1	-.152	-.443	-.771	.014	-.245	.081	.091
	Sig. (2-tailed)	.383	.940	.919	.755	.182	.216		.903	.708	.440	.991	.843	.949	.942
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
H	Pearson Correlation	-.434	-.998*	-1.000*	-.973	.132	.472	-.152	1	-.819	.747	.986	.996	.973	-.998*
	Sig. (2-tailed)	.714	.038	.017	.148	.916	.687	.903		.389	.463	.106	.060	.149	.040
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
I	Pearson Correlation	.873	.851	.834	.665	-.677	.120	-.443	-.819	1	-.230	-.903	-.761	-.929	.853
	Sig. (2-tailed)	.325	.352	.373	.537	.526	.924	.708	.389		.852	.283	.449	.241	.350
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
J	Pearson Correlation	.275	-.706	-.729	-.880	-.560	.939	-.771	.747	-.230	1	.626	.806	.573	-.704
	Sig. (2-tailed)	.823	.501	.480	.315	.622	.224	.440	.463	.852		.569	.403	.612	.503
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
K	Pearson Correlation	-.578	-.994	-.990	-.921	.295	.319	.014	.986	-.903	.626	1	.966	.998*	-.994
	Sig. (2-tailed)	.608	.069	.090	.254	.809	.793	.991	.106	.283	.569		.167	.042	.067
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
L	Pearson Correlation	-.347	-.988	-.993	-.991	.038	.553	-.245	.996	-.761	.806	.966	1	.947	-.988
	Sig. (2-tailed)	.774	.098	.077	.088	.976	.627	.843	.060	.449	.403	.167		.209	.100
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
M	Pearson Correlation	-.631	-.985	-.979	-.894	.358	.255	.081	.973	-.929	.573	.998*	.947	1	-.985
	Sig. (2-tailed)	.565	.111	.132	.296	.767	.836	.949	.149	.241	.612	.042	.209		.109
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3
N	Pearson Correlation	.489	1.000**	.999*	.957	-.194	-.416	.091	-.998*	.853	-.704	-.994	-.988	-.985	1
	Sig. (2-tailed)	.675	.002	.023	.187	.876	.727	.942	.040	.350	.503	.067	.100	.109	
	N	3	3	3	3	3	3	3	3	3	3	3	3	3	3

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

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Bayessian Regression Technique For Modeling Multicolinear Data

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Abstract: *This paper examined the behaviours of two frequentist regression methods (the ridge regression and ordinary least squares (OLS)) and Bayesian linear regression method on data with inherent collinear structure. Data sets with reasonable degrees of multicollinearity at some selected sample sizes were simulated. The three regression types were fitted to various data and the performances of both the ridge and OLS estimators were compared with that of the Bayesian linear regression estimators using Normal-Gamma conjugate prior. The goal is to examine the relative efficiency of the Bayesian estimator, which integrates some prior information with the information available in the data in its regression estimation, over the two frequentist regression techniques. Results from Monte Carlo studies established the supremacy of the Bayesian estimators over both the OLS and the ridge estimators. Although, the ridge regression estimators expectedly performs better than the OLS estimators given the degree of multicollinearity in the simulated data, the results generally showed that Bayesian linear regression estimator is relatively more efficient (with smaller mean square errors) than the two frequentist regression techniques given the same data structure.*

Keywords*: Bayessian, Modeling, Multicolinear, Regression, and Simulation*

1.0 Introduction

In statistical inference, there are two broad categories of interpretations of probability: Bayesian inference (Byron Hall, STATISTICAT, LLC, Bayesian Inference Article p. 1-2) and frequentist inference (Byron Hall, STATISTICAT, LLC, and Bayesian Inference Article). These views often differ with each other on the fundamental nature of probability. Frequentist inference loosely defines probability as the limit of an event's relative frequency in a large number of trials, and only in the context of experiments that are random and well-defined. Bayesian inference, on the other hand, is able to assign probabilities to any statement, even when a random process is not involved. In Bayesian inference, probability is a way to represent an individual's degree of belief in a statement, or given evidence. Within Bayesian inference, there are also different interpretations of probability, and different approaches evolved based on those interpretations. The most popular interpretations and approaches are objective Bayesian inference (Berger 2006) and subjective Bayesian inference (Anscombe and Aumann

1963; *Bernardo* 2008). Objective Bayesian inference is often associated with Bayes and Price (1763), Laplace (1814), and *Roberts* (2007). Subjective Bayesian inference is often associated with Ramsey (1926), *Simon*, (2009), and *Bernardo and Smith, (2000)*.

1.1 Bayes' Theorem

Bayes' theorem shows the relation between two conditional probabilities that are the reverse of each other. This theorem is named after Reverend Thomas Bayes (1702-1761), and is also referred to as Bayes' law or Bayes' rule (Bayes and Price 1763). Bayes' theorem expresses the conditional probability, or `posterior probability', of an event A after B is observed in terms of the 'prior probability' of A, prior probability of B, and the conditional probability of B given A. Bayes' theorem is valid in all common interpretations of probability.

When no data are available, a *prior distribution* is used to quantify our knowledge about the parameter. When data are available, we can update our prior knowledge using the conditional distribution of parameters, given the data. The transition from the prior to the posterior is possible via the Bayes theorem. Suppose that before the experiment our prior distribution describing Aisπ (*A*)*:* The data are coming from the assumed model (likelihood) which depends on the parameter and is denoted by *f(x/A).*Bayes theorem updates the prior,$\pi\ (A)$ to the posterior by accounting for the data *x* through the relationship.

$$\pi(A/x) = \frac{f(x/A)\pi(A)}{m(x)} \qquad (1.1)$$

Where *m*(*x*) is a normalizing constant, *m*(*x*) =$\int f(x/A)\pi(A)\ dA$ (1.2)

Once the data *x* are available, *A* is the only unknown quantity and the posterior distribution π(*A/x*) completely describes the uncertainty. There are two key advantages of Bayesian paradigm: (i) once the uncertainty is expressed via the probability distribution and the statistical inference can be automated, it follows a conceptually simple recipe, and (ii) available prior information is coherently incorporated into the statistical model.

1.2 Bayesian Linear Regression

In statistics, **Bayesian linear regression** is an approach to linear regression in which the statistical analysis is undertaken within the context of Bayesian inference. When the regression model has errors that have a normal distribution, and if a particular form of prior distribution is assumed, explicit results are available for the posterior probability distributions of the model's parameters. Consider a standard linear regression problem, in which for$i = 1, \dots, n$ we specify the conditional distribution of$y_i$given a *1×k* predictor vector$\boldsymbol{x_i}$:

$$y_i = \boldsymbol{x_i}^{\boldsymbol{T}}\boldsymbol{\beta} + \epsilon_i \qquad (1.3)$$

where $\boldsymbol{\beta}$ is a*k×1* vector of regression parameters to be estimated, and the$\epsilon_i$is independent and identically-distributed and normally distributed random error of the model with $\epsilon_i \sim N(0, \sigma^2)$.
From the regression model in (1.2), the following likelihood function is developed within the Bayesian concept:

$$p(\boldsymbol{y}|\boldsymbol{X}, \boldsymbol{\beta}, \sigma^2) \propto (\sigma^2)^{\frac{-n}{2}}\, exp\left(-\frac{1}{2(\sigma^2)}(\boldsymbol{y} - \boldsymbol{X\beta})^{\boldsymbol{T}}(\boldsymbol{y} - \boldsymbol{X\beta})\right) \qquad (1.4)$$

The ordinary least squares solution is to estimate the coefficient vector using the Moore-Penrose pseudo-inverse:

$$\hat{\beta} = (X^T X)^{-1}(X^T Y) \qquad (1.5)$$

Where **X** is the ***n×k*** design matrix of predictor variables, each row of which is a predictor vector$\boldsymbol{x_i}^{\boldsymbol{T}}$; and **y**is the column n-vector$(\boldsymbol{y_{1,}} \dots, \boldsymbol{y_n})^{\boldsymbol{T}}$.
This is a frequentist approach, and it assumes that there are enough measurements (samples) to say something meaningful about $\boldsymbol{\beta}$. In the Bayesian approach, the data are supplemented with additional information in the form of a prior probability distribution. The prior belief about the parameters is combined with the data's likelihood function according to Bayes theorem to yield the posterior belief about the parameters $\boldsymbol{\beta}$ andσ. The prior can take different functional forms depending on the domain and the information that is available a priori.

2.1 Linear Regression Model

The linear regression model is use to study the relationship between a dependent variable and one or more independently variables. The generic form of the linear regression model is

$$y = f(x_1, x_2, \; . \; . \; . \; , \; x_k) + \varepsilon \qquad (2.1)$$
$$= \beta_o + \beta_1 x_1 + \beta_2 x_2 + \; . \; . \; . + \beta_k x_k \; + \varepsilon \qquad (2.2)$$

Where y is the dependent variable or explained variable and $x_1, \dots, x_k$ are the independent or explanatory variables. The function$f(x_1, \dots, x_k)$ is commonly called population regression equation of y on$x_1, \dots, x_k$. In this setting, y is the regress andx_k, k=1 … K, are the regressors or covariates. The term ε is a random disturbance so named because it “disturbs” an otherwise stable relationship. (Greene, 2000)

2.1.2 Normal Linear Regression Model

The above equation can be expressed in matrix form as a normal linear regression model and thus the model is given as:

$$y = X\beta + \epsilon \qquad (2.3)$$

Where $y = (y_{1,} \dots, y_n)^T, \beta = (\beta_1, \dots, \beta_j)^T$and assuming initially that $\epsilon \sim N(0, \sigma^2)$
The addition of the assumption of normality of ϵ leads to normal linear regression model. (Gujarati (2004).

2.1.3 Linear Regression Model in Matrixnotation

Suppose we have data on a dependent variable, y_i, and k explanatory variables $x_{i1}, \dots, x_{ik}$ for $i = 1, \dots, n$. The linear regression model is given by:

$$y_i = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i \qquad (2.4)$$

The above notation is such that x_{i1} is implicitly set to 1 to allow for an intercept. This model can be written more compactly in matrix notation by defining the $n \times 1$ vectors:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix}$$

and

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ . \\ . \\ . \\ \epsilon_n \end{bmatrix}$$

the $k \times 11$

=β_1β_2...β_k
and the $n \times k$ matrix

$$X = \begin{bmatrix} 1 x_{12} \cdots x_{1k} \\ 1 x_{22} \cdots x_{2k} \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ 1 x_{n2} \cdots x_{nk} \end{bmatrix}$$

And writing

$$y = X\beta + \epsilon \qquad (2.5)$$

Using the definition of matrix multiplication it can be verified that (2) is equivalent to the n equations defined by (1) (Koop. 2003),

3.0 Methodology

3.1 Bayesian Method

The paper presents the posterior distribution of the Bayesian normal linear regression, properties of the parameters of interest with respect to their credible intervals and highest posterior densities.

3.1.1 The Posterior Distribution of Bayesian Normal Linear Regression

The posterior distribution $p(\beta, h|y)$ defined as:

$$p(\beta, h|y) \propto \ p(\beta, h) \times p(y|\beta, h) \tag{3.1}$$

Where $p(\beta, h) = \frac{h^{\frac{v_0+k}{2}-1}}{2\pi^{\frac{k}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\left\{exp\left[\frac{-h}{2}(\beta-\beta_0)^T(\Sigma_0)^{-1}(\beta-\beta_0)+\frac{v_0}{s_0^{-2}}\right]\right\}$(3.2)

And

$$p(y|\beta, h) = \frac{h^{\frac{n}{2}}}{(2\pi)^{\frac{n}{2}}}\left\{exp\left[\frac{-h}{2}(vs^2+(b-\beta)^T(X^TX)(b-\beta))\right]\right\} \tag{3.3}$$

Thus

$$p(\beta, h|y) \propto \frac{h^{\frac{v_0+k}{2}-1}}{2\pi^{\frac{k}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\left\{exp\left[\frac{-h}{2}(\beta-\beta_0)^T(\Sigma_0)^{-1}(\beta-\beta_0)+\frac{v_0}{s_0^{-2}}\right]\right\}\times$$

$$\frac{h^{\frac{n}{2}}}{(2\pi)^{\frac{n}{2}}}\left\{exp\left[\frac{-h}{2}(vs^2+(b-\beta)^T(X^TX)(b-\beta))\right]\right\} \tag{3.4}$$

Let $c = \frac{1}{(2\pi)^{\frac{k+n}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}$

Then

$$p(\beta, h|y) \propto h^{\frac{v_0+n+k}{2}-1}\left\{exp\left[\frac{-h}{2}\left((\beta-\beta_0)^T(\Sigma_0)^{-1}(\beta-\beta_0)+\frac{v_0}{s_0^{-2}}+vs^2+(b-\beta)^T(X^TX)(b-\beta)\right)\right]\right\} \tag{3.5}$$

Expanding the terms within the exponent bracket, we have:

$$(\beta-\beta_0)^T(\Sigma_0)^{-1}(\beta-\beta_0)+(b-\beta)^T(X^TX)(b-\beta) = \tag{3.6}$$

$$\beta^T(\Sigma_0)^{-1}\beta-\beta^T(\Sigma_0)^{-1}\beta_0-{\beta_0}^T(\Sigma_0)^{-1}\beta+{\beta_0}^T(\Sigma_0)^{-1}\beta_0+b^T(X^TX)b-b^T(X^TX)\beta-\beta^T(X^TX)b+\beta^T(X^TX)\beta= \tag{3.7}$$

$$\beta^T[(\Sigma_0)^{-1}+X^TX]\beta-\beta^T[(\Sigma_0)^{-1}\beta_0+(X^TX)b]-[{\beta_0}^T(\Sigma_0)^{-1}+b^T(X^TX)]\beta+{\beta_0}^T(\Sigma_0)^{-1}\beta_0+b^T(X^TX)b = \tag{3.8}$$

If we let

$$\beta^* = \Sigma^*({\Sigma_0}^{-1}\beta_0+X^TXb)$$

$$\Sigma^* = ({\Sigma_0}^{-1}+X^TX)^{-1}$$

$$\beta^T(\Sigma^*)^{-1}\beta-\beta^T(\Sigma^*)^{-1}\Sigma^*[(\Sigma_0)^{-1}\beta_0+(X^TX)b]-(\Sigma^*)^{-1}\Sigma^*[{\beta_0}^T(\Sigma_0)^{-1}+b^T(X^TX)]\beta+{\beta_0}^T(\Sigma_0)^{-1}\beta_0+b^T(X^TX)b =$$

$$\beta^T(\Sigma^*)^{-1}\beta-\beta^T(\Sigma^*)^{-1}\beta^*-{\beta^*}^T(\Sigma^*)^{-1}\beta+{\beta_0}^T(\Sigma_0)^{-1}\beta_0+b^T(X^TX)b$$

Simplifying further:

$$\beta^T(\Sigma^*)^{-1}\beta - \beta^T(\Sigma^*)^{-1}\beta^* - \beta^{*T}(\Sigma^*)^{-1}\beta + \beta_0{}^T(\Sigma_0)^{-1}\beta_0 + b^T(X^TX)b =$$
$$\beta^T(\Sigma^*)^{-1}\beta - \beta^T(\Sigma^*)^{-1}\beta^* - \beta^{*T}(\Sigma^*)^{-1}\beta^* + \beta^{*T}(\Sigma^*)^{-1}\beta^* - \beta^{*T}(\Sigma^*)^{-1}\beta + \beta_0{}^T(\Sigma_0)^{-1}\beta_0 + b^T(X^TX)b =$$
$$(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*) - \beta^{*T}(\Sigma^*)^{-1}\beta + \beta_0{}^T(\Sigma_0)^{-1}\beta_0 + b^T(X^TX)b \quad (3.9)$$
The last three terms can be further combined to yield:

$$\beta_0{}^T(\Sigma_0)^{-1}\beta_0 + b^T(X^TX)b - \beta^{*T}(\Sigma^*)^{-1}\beta = (b-\beta_0)^T\left[\Sigma_0{}^{-1} + (X^TX)^{-1}\right]^{-1}(b-\beta_0) \quad (3.10)$$

Thus the posterior can be written as:

$$p(\beta,h|y) \propto h^{\frac{v_0+n+k}{2}-1}\left\{exp\left[\frac{-h}{2}\Big((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*) + v_0s_0{}^2 + vs + (b-\beta_0)^T\left[\Sigma_0{}^{-1}+(X^TX)^{-1}\right]^{-1}(b-\beta_0)\Big)\right]\right\} =$$
$$\propto h^{\frac{v_0+n+k}{2}-1}\left\{exp\left[\frac{-h}{2}((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*))\right]\right\} * exp\left[\frac{-h}{2}\Big(v_0s_0{}^2 + vs + (b-\beta_0)^T\left[\Sigma_0{}^{-1}+(X^TX)^{-1}\right]^{-1}(b-\beta_0)\Big)\right]$$

$$\propto h^{\frac{k}{2}}\left\{exp\left[\frac{-h}{2}((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*))\right]\right\} * h^{\frac{v*}{2}-1}exp\left[\frac{-hv^*}{2s^{-2*}}\right] \quad (3.12)$$

The above is indeed the kernel of a Normal-gamma distribution.

Therefore

$$\beta,h|y \sim NG(\beta^*, \Sigma^*, v^*, s^{-2^*}) \quad (3.13)$$

Where

$$\beta^* = \Sigma^*(\Sigma_0{}^{-1}\beta_0 + X^TXb) \quad (3.14)$$
$$\Sigma^* = (\Sigma_0{}^{-1} + X^TX)^{-1} \quad (3.15)$$
$$v^* = v_0 + n \quad (3.16)$$
$$s^{-2^*} = \frac{v^*}{v_0s_0{}^2 + vs + (b-\beta_0)^T(\Sigma_0{}^{-1} + (X^TX)^{-1})^{-1}(b-\beta_0)} \quad (3.17)$$
$$v^*s^{2*} = v_0s_0{}^2 + vs + (b-\beta_0)^T(\Sigma_0{}^{-1} + (X^TX)^{-1})^{-1}(b-\beta_0) \quad (3.18)$$

3.1.2 Properties of the posterior parameters

3.1.2.1 Marginal distribution of $\boldsymbol{\beta}$

In this setting h is not of immediate interest and is therefore considered as nuisance parameter. It follows that h has to be integrated out to get the marginal distribution of $\boldsymbol{\beta}^*$:

$$p(\beta|y) = \int_0^\infty p(\beta,h|y)d(h) \quad (3.19)$$
$$= \int_0^\infty \frac{1}{(2\pi)^{\frac{k+n}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0{}^{-2}}{v_0}\right)^{\frac{v_0}{2}}} h^{\frac{v_0+n+k}{2}-1}\left\{exp\left[\frac{-h}{2}\Big((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*) + v_0s_0{}^2 + vs + (b-\beta_0)^T\left[\Sigma_0{}^{-1}+(X^TX)^{-1}\right]^{-1}(b-\beta_0)\Big)\right]\right\} dh$$

$$= \frac{1}{(2\pi)^{\frac{k+n}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\int_0^\infty h^{\frac{v_0+n+k}{2}-1}\left\{exp\left[\frac{-h}{2}\left((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v_0s_0^2+vs\right.\right.\right.$$

$$\left.\left.\left.+(b-\beta_0)^T\left[\Sigma_0^{-1}+(X^TX)^{-1}\right]^{-1}(b-\beta_0)\right)\right]\right\}dh$$

$$= \frac{1}{(2\pi)^{\frac{k+n}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\int_0^\infty h^{\frac{v*+k}{2}-1}\left\{exp\left[\frac{-h}{2}((\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*})\right]\right\}dh$$

(3.20)

Using integration by substitution

Let $m = \frac{h(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}{2}$ then

$$\frac{dm}{dh} = \frac{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}{2}$$

Therefore:

$$= c\int_0^\infty\{exp(-m)\}\left(\frac{2m}{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}\right)^{\frac{v^*+k}{2}-1}\frac{2dm}{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}$$

$$= c\left(\frac{2}{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}\right)^{\frac{v^*+k}{2}}\left[\int_0^\infty m^{\frac{v^*+k}{2}-1}\{exp(-m)\}dm\right]$$

Recall from gamma function that

$$\left[\int_0^\infty m^{\frac{v^*+k}{2}-1}\{exp(-m)\}dm\right] = \Gamma_{\frac{v^*+k}{2}}$$

Thus

$$= c\left(\frac{2}{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}\right)^{\frac{v^*+k}{2}}\Gamma_{\frac{v^*+k}{2}}$$

$$= \frac{\Gamma_{\frac{v^*+k}{2}}}{(2\pi)^{\frac{k+n}{2}}|\Sigma_0|^{\frac{1}{2}}\Gamma_{\frac{v_0}{2}}\left(\frac{2s_0^{-2}}{v_0}\right)^{\frac{v_0}{2}}}\left(\frac{(\beta-\beta^*)^T(\Sigma^*)^{-1}(\beta-\beta^*)+v^*s^{2*}}{2}\right)^{-\frac{v^*+k}{2}} \quad (3.21)$$

Hence $p(\boldsymbol{\beta}|\boldsymbol{y})$follows the multivariate t-distribution defines as follows:

$$(\boldsymbol{\beta}|\boldsymbol{y})\sim\boldsymbol{t}(\beta^*, s^{2*}\Sigma^*, v^*) \quad (3.22)$$

$$\boldsymbol{E}(\beta) = \beta^* \quad (3.23)$$

$$\boldsymbol{var}(\beta) = \frac{v^*s^{2*}}{v^*-2}\Sigma^* \quad (3.24)$$

3.1.2.2 Marginal distribution of h

To derive the marginal posterior density for *h*, we can use the "reversed" version of bayes rule:

$$p(h|y) = \frac{p(\beta,h|y)}{p(\beta|h,y)} \quad (3.25)$$

The above follow from the definition of a Normal-gamma distribution, which is a product of a conditional Normal distribution, and gamma distribution.

From the posterior definition:

$$\beta, h|y \sim NG(\beta^*, \Sigma^*, v^*, s^{-2^*})$$

One can easily define the conditional normal distribution of βas:

$$\beta|h, y \sim N(\beta^*, h^{-1}\Sigma^*) \quad (3.26)$$

Therefore

$$p(\beta|h, y) \propto h^{\frac{k}{2}} exp\left\{\frac{-h}{2}((\beta - \beta^*)^T(\Sigma^*)^{-1}(\beta - \beta^*))\right\} \quad (3.27)$$

Thus

$$p(h|y) \propto h^{\frac{v*+k}{2}-1}\left\{exp\left[\frac{-h}{2}((\beta - \beta^*)^T(\Sigma^*)^{-1}(\beta - \beta^*) + v^*s^{2*})\right]\right\} * \left\{exp\frac{-h}{2}((\beta - \beta^*)^T(\Sigma^*)^{-1}(\beta - \beta^*))\right\}^{-1}$$

$$\propto h^{\frac{k}{2}}\left\{exp\left[\frac{-h}{2}((\beta - \beta^*)^T(\Sigma^*)^{-1}(\beta - \beta^*))\right]\right\} * h^{\frac{v*}{2}-1}exp\left[\frac{-hv^*}{2s^{-2*}}\right] * \left\{h^{\frac{k}{2}}exp\frac{-h}{2}((\beta - \beta^*)^T(\Sigma^*)^{-1}(\beta - \beta^*))\right\}^{-1}$$

$$p(h|y) \propto h^{\frac{v*}{2}-1}exp\left[\frac{-hv^*}{2s^{-2*}}\right] \quad (3.28)$$

Hence h follows a Gamma distribution define as$h|y \sim G(s^{-2*}, v^*)$ (3.29)

$E(h) = s^{-2*}$ (3.30)

$$var(h|y) = \frac{2(s^{-2*})^2}{v^*} \quad (3.31)$$

The above definition follows from gamma distribution with parameters v^* degree of freedom and mean s^{-2*}.

3.1.2.3 Interpretation of the Estimators

β is now the posterior mean for β, which is the Bayesian estimator for the unknown regression coefficient and thus interpreted as the weighted average of the prior mean β_0 and OLS estimator b where the weight reflect the strength of information by prior$(\Sigma_0)^{-1}$ and data $X^T X$. The latter of these reflects the confidence in the prior. For instance, if the prior variance selected is high, that implies we are very uncertain about what likely values of β are. As a result, $(\Sigma_0)^{-1}$will be small and little weight will be attached to β_0; the best prior guess at what $\beta$is. The term $X^T X$plays a similar role with respect to databased information. Loosely speaking, it reflects the degree of confidence that the data have in its best guess for β; the OLS estimate b. According to frequentist econometrics, we recognize$(X^T X)^{-1}$ as being proportional to the variance ofβ.Note that, for both prior mean and the OLS estimate, the posterior mean attaches weight proportional to their precisions (i.e. the inverse of their variances). Hence, Bayesian methods combine data and prior information in a sensible way.

In frequentist econometrics, the variance of the OLS estimator for the regression model given in (2.9) is $s^2(X^T X)^{-1}$. The Bayesian analogue is the posterior variance of βgiven in (3.24), which has a very similar form, but incorporates both prior and data information. For instance, (3.14) can be informally interpreted as saying "posterior precision is an average of prior precision $(\sum_0)^{-1}$and data precisionX^TX . Similarly, (3.18) has an intuitive interpretation of posterior sum of squared errors (v^*s^{2*}) is the sum of prior sum of squared errors $(v_0{s_0}^2)$, OLS sum of squared errors (vs), and a term which measures the conflictbetween prior and data information".

The other equations above also emphasize the intuition that the Bayesian posterior combines data and prior information. Furthermore, the natural conjugate prior implies that the prior can be interpreted as arising from a fictitious dataset (e.g. v and$n$play the same role in (3.16) and (3.18) and, hence,$v$can beinterpreted as a prior sample size).

It is useful to draw out the similarities, differences between what a Bayesian would do, and what a frequentist would do. The latter might calculate b and its variance, $s^2(X^TX)^{-1}$, andestimate σ^2 by s^2. The former might calculate the posterior mean and variance ofβ (i.e. $\beta^*$and $\frac{v^*s^{2*}}{v^*-2}\sum^*$) and estimate $h = \sigma^{-2}$ by its posterior mean,s^{-2*} . These arevery similar strategies, except for two important differences. First, the Bayesian formulae all combine prior and data information. Secondly, the Bayesian interpretsβ as a random variable, whereas the frequentist interprets b as a randomvariable.

The fact that the natural conjugate prior implies prior information enters inthe same manner as data information helps with prior elicitation. For instance,when choosing particular values for$\beta_0, \sum_{\mathbf{0}}, {s_0}^2$ and $v_0$it helps to know that β_0 isequivalent to the OLS estimate from an imaginary data set of v_0 observations withan imaginaryX^TXequal to$(\sum_{\mathbf{0}})^{-\mathbf{1}}$ and an imaginary s^2 given by${s_0}^2$.

4.0 Results and Discussion

4.1 Results

The variance inflation factor for all the variables is as follows;

$$Vif(X_1) = 5.807767, Vif(X_2) = 109.514563, Vif(X_3) = 92.23301$$

Table 4.1: Summary of estimate of coefficient and standard error at sample size 20 - 120

Sample size	BAYES ESTIMATE			OLS ESTIMATE		RIDGE ESTIMATE	
		coef	Std deviation	coef	Std Error	coef	Std Error
$n =$ 2020	$\beta_0 = 25$	**25**	**7.1**	698.15	20676.02	9502.78	100.34
	$\beta_1 = 165$	**164.96**	**2.34**	164.91	3.17	164.48	2.82
	$\beta_2 = 150$	**150.06**	**1.16**	150.47	12.45	155.68	1.36
	$\beta_3 = 345$	**345.02**	**1.26**	344.72	9.41	340.67	1.51
$n =$ 4040	$\beta_0 = 25$	**25**	**10.08**	-264.07	18962.12	6063.21	131.09
	$\beta_1 = 165$	**165**	**2.36**	165.02	2.91	164.71	2.59
	$\beta_2 = 150$	**150.04**	**1.15**	149.87	11.42	153.62	1.25

	$\beta_3 = 345$	**345**	**1.27**	345.13	8.63	342.22	1.39
$n =$ 6060	$\beta_0 = 25$	**25**	**8.53**	447.47	12432.24	5426.41	109.99
	$\beta_1 = 165$	**165.08**	**1.63**	165.06	1.9	164.81	1.7
	$\beta_2 = 150$	**149.99**	**0.79**	150.24	7.49	153.19	0.82
	$\beta_3 = 345$	**344.95**	**0.88**	344.76	5.66	342.47	0.91
$n =$ 8080	$\beta_0 = 25$	**25**	**9.56**	-604.39	12234.45	4043.53	116.41
	$\beta_1 = 165$	**165.07**	**1.59**	165.12	1.87	164.89	1.67
	$\beta_2 = 150$	**149.97**	**0.77**	149.59	7.37	152.34	0.81
	$\beta_3 = 345$	**344.96**	**0.85**	345.24	5.57	343.11	0.9
$n =$ 10000	$\beta_0 = 25$	**25**	**8.25**	5.39	9349.61	3823.23	108.36
	$\beta_1 = 165$	**164.95**	**1.22**	164.95	1.43	164.76	1.28
	$\beta_2 = 150$	**149.99**	**0.59**	149.97	5.63	152.23	0.62
	$\beta_3 = 345$	**345.03**	**0.66**	345.04	4.25	343.28	0.69
$n =$ 12020	$\beta_0 = 25$	**25**	**7.75**	-76.8	7910.44	3645.4	94.15
	$\beta_1 = 165$	**165.04**	**1.05**	165.05	1.21	164.87	1.08
	$\beta_2 = 150$	**149.99**	**0.51**	149.93	4.76	152.13	0.52
	$\beta_3 = 345$	**344.98**	**0.56**	345.02	3.6	343.31	0.58

Table 4.2: Summary of estimate of coefficient and standard error at sample size 140 –200 500, 1000.

Sample size	**BAYES ESTIMATE**			**OLS ESTIMATE**		**RIDGE ESTIMATE**	
		Coef	Std deviation	Coef	Std error	Coef	Std error
$n =$ 14040	$\beta_0 = 25$	**25**	**9.71**	-113.28	9193.91	3325.15	118.49
	$\beta_1 = 165$	**164.99**	**1.22**	165	1.41	164.83	1.25
	$\beta_2 = 150$	**150.01**	**0.59**	149.92	5.54	151.96	0.61
	$\beta_3 = 345$	**345**	**0.65**	345.07	4.18	343.48	0.68
$n =$ 16060	$\beta_0 = 25$	**25**	**9.03**	166.19	7929.53	3244.19	114.06
	$\beta_1 = 165$	**165.02**	**1.06**	165.01	1.21	164.86	1.08
	$\beta_2 = 150$	**150.02**	**0.51**	150.11	4.78	151.93	0.53
	$\beta_3 = 345$	**344.99**	**0.57**	344.92	3.61	343.51	0.58
$n =$ 18080	$\beta_0 = 25$	**25**	**9.18**	-206.9	7612.49	2768.65	113.31
	$\beta_1 = 165$	**164.99**	**1.02**	165.01	1.17	164.86	1.04
	$\beta_2 = 150$	**150**	**0.49**	149.86	4.59	151.63	0.51
	$\beta_3 = 345$	**345**	**0.55**	345.11	3.46	343.74	0.56

$n =$ 20000	$\beta_0 = 25$	**25**	**8.82**	-227.56	6931.2	2727.32	103.92
	$\beta_1 = 165$	**165.02**	**0.93**	165.04	1.06	164.89	0.95
	$\beta_2 = 150$	**150.02**	**0.45**	149.86	4.17	151.61	0.46
	$\beta_3 = 345$	**344.99**	**0.5**	345.1	3.15	343.74	0.51
$n =$ 50000	$\beta_0 = 25$	**25**	**8.96**	25	4393.89	1893.03	110.45
	$\beta_1 = 165$	**165.02**	**0.6**	165.02	0.67	164.92	0.6
	$\beta_2 = 150$	**149.99**	**0.29**	149.99	2.65	151.1	0.3
	$\beta_3 = 345$	**344.99**	**0.32**	344.99	2	344.13	0.33
$n =$ 100000	$\beta_0 = 25$	**25**	**9.06**	25	3134.79	1311.35	103.97
	$\beta_1 = 165$	**164.98**	**0.43**	164.98	0.48	164.91	0.43
	$\beta_2 = 150$	**150**	**0.21**	150	1.89	150.76	0.22
	$\beta_3 = 345$	**345.01**	**0.23**	345.01	1.43	344.42	0.23

Table 4.3: Summary of 95% confidence and credible interval for the estimators

Sample size	**BAYES 95% CREDIBLE INTERVAL**			**OLS 95% CONFIDENCE INTERVAL**		**RIDGE 95% CONFIDENCE INTERVAL**	
		Lower	Upper	Lower	Upper	Lower	Upper
$n =$ 2020	$\beta_0 = 25$	**14.16597**	**36.17456**	-63301.8	24360.65	9290.082	9715.488
	$\beta_1 = 165$	**159.1038**	**168.078**	157.4136	170.8445	158.5006	170.4588
	$\beta_2 = 150$	**145.903**	**150.4816**	110.5315	163.3343	152.7959	158.5682
	$\beta_3 = 345$	**343.4001**	**348.1928**	335.1006	374.9946	337.4551	343.8773
$n =$ 6060	$\beta_0 = 25$	**10.71535**	**42.39604**	-48860.8	948.7104	5206.075	5646.74
	$\beta_1 = 165$	**163.8568**	**169.2768**	165.0413	172.6727	161.4142	168.2089
	$\beta_2 = 150$	**149.3212**	**151.8048**	121.1434	151.1458	151.5473	154.8344
	$\beta_3 = 345$	**342.6347**	**345.5541**	343.2246	365.8922	340.6474	344.3002
$n =$ 10000	$\beta_0 = 25$	**10.76345**	**40.3885**	-14766.5	22351.19	3608.133	4038.33
	$\beta_1 = 165$	**162.28**	**167.3016**	161.7387	167.4256	162.2273	167.2907
	$\beta_2 = 150$	**149.4397**	**151.6747**	141.4799	163.8375	151.0066	153.4618
	$\beta_3 = 345$	**343.762**	**346.4126**	334.9215	351.8133	341.9207	344.6457
$n =$ 20000	$\beta_0 = 25$	**7.558032**	**43.71414**	-16502.7	10835.94	2522.372	2932.274
	$\beta_1 = 165$	**163.5901**	**167.1153**	163.4495	167.6381	163.0267	166.7561
	$\beta_2 = 150$	**149.1336**	**150.986**	140.0205	156.4877	150.7072	152.5223
	$\beta_3 = 345$	**343.8519**	**345.7796**	339.8815	352.323	342.7376	344.748

Table 4.4: Mean square error of the estimators at various sample sizes

Sample Sizes	BAYES MSE	OLS MSE	RIDGE MSE
20	**3.33**	1.32×10^{8}	67235991
40	**1.62**	60055660	30204822
60	**1.11**	40863372	22146422
80	**0.86**	29947007	13710192
100	**0.71**	22829325	11558297
120	**0.55**	21264974	9871460
140	**0.48**	17336409	8464848
160	**0.42**	15317101	7756157
180	**0.38**	12908635	6250229
200	**0.34**	12767788	6159415
500	**0.14**	5109822	2668017
1000	**0.07**	2603927	1290459

Table 4.5: Mean square error of prediction for the estimators at various sample sizes

Sample Sizes	BAYES MSEP	OLS MSEP	RIDGE MSEP
20	**7213.086**	7303.68	8029.317
40	**4635.332**	4650.295	4522.359
60	**8911.436**	8919.751	9087.504
80	**12461.58**	12468.18	12288.64
100	**11809.74**	11802.38	12185.98
120	**11360.94**	11369.32	11229.35
140	**11345.92**	11338.87	11496.92
160	**10553.39**	10551.4	10511.69
180	**13365.38**	13364.45	13314.26
200	**11596.22**	11586.63	11464.2

4.2 Discussion of Results

In this research, two frequentist methods (Ordinary Least Square (OLS) & Ridge Regression (RR)) and Bayesian Regression were used to fit a set of collinear data. OLS performed as expected due to the effect of collinearity existing between the predictors, RR also produces a fairly precise estimate but the estimates were totally different from the true value (Bias). The Bayesian regression which uses the prior as an ingredient to solve the problem of collinearity produces the closest estimates to the true value and also precise. The above results can be found in table 4.1. Increasing the sample size in table 4.2 improves the estimate of the three estimators, OLS estimates and Bayesian estimates now converges to be the same, RR and Bayesian precision also converges to be the same. Table 4.3 presents the 95% confidence and credible interval as in the case of frequentist and Bayesian respectively. Due to the imprecise standard errors of the frequentists estimate, the OLS and RR produces wide confidence intervals i.e if we are to test the hypothesis of significance of β_o, the hypothesis of nonsignificance would not be rejected. The Bayesian credible intervals interpreted as the probability that the unknown parameter will fall in the interval were narrower compared to the frequentist counterparts.

Furthermore, Table 4.4 presents the Mean Square Error MSE which is used to measure the average closeness of the estimators to the true values, it was observed that the Bayesian estimator produce << lower MSE compared to OLS and RR. Although RR MSE is approximately half of the OLS MSE, but its better is not the best.

To access the predictive ability of the estimators, Mean Square Error of Prediction was used, the results were presented in table 4.5. It was observed that the three estimators performed extremely the same, with slight better performance from the Bayesian estimators in some cases. All the results discussed above were also confirmed using box & whisker plots and line graphs.

5.2 Conclusion

In this study a simple way of modelling collinear data under simulation approach was presented. It was observed that modelling collinearity in a full Bayesian using a Normal-Gamma conjugate prior have improved the precision of the estimates and the efficiency of the inferences about the parameters.

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A Review of Mathematical Modelling of Thin-Layer Sun Drying of Agricultural Products

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Abstract: *The drying of agricultural products is a complex operation that demands much energy and time. In practice, drying increases product shelf-life and reduces the bulk and weight of the product, thus simplifying transport. Occasionally, drying may lead to a great decrease in the volume of the product, leading to a decrease in storage space requirements. Studies have shown that dependence purely on experimental drying practices, without mathematical considerations of the drying kinetics, can significantly affect the efficiency of drying, increase the cost of production, and reduce the quality of the dried product. Present work involves the study of thin layer drying characteristics of different agricultural products under open sun drying process. The drying data fitted into different thin layer drying models. The performance of these models was investigated by comparing the coefficient of determination (R^2), reduced chi-square(X^2) and root mean square error (RMSE) between the observed and predicted moisture ratio. On the basis of highest value of correlation coefficient (R) and coefficient of determination (R^2) and lowest value of reduced chi- square(X^2) and root mean square error (RMSE) appropriate model will be selected.*

Keywords*: Agricultural, Drying, Modeling, Layer, and Product*

1.0 INTRODUCTION

Food is the basic necessity requires by human beings for comfortability of living. Food production and consumption imbalance is the major problem faced by human beings. Increase of food supply and limitation of population growth are cited as two solutions for the imbalance of food. Although, both the solutions require considerable amount of capital and time to achieve (Murthy, 2009). Another solution to the food production and consumption imbalance is the reduction in the postharvest losses occurring in the developing countries. Therefore, the only method to reduce the postharvest losses is through food preservation, and drying is the method that is being adopted since many centuries ago (Blaise *et al.,* 2009; Murthy, 2009).

Therefore, drying can be defined as a thermo-physical process (Silva *et al.,* 2014) that consists of reduction or removal of moisture content from a wet material by the application of heat energy (Cakmak and Yildiz, 2012; Chatta *et al.,* 2018; El-Sebaii and Shalaby, 2012; Gulcimen and Karakaya, 2016; Polatoğlu and Beşe, 2017). Drying of agricultural products is one of the oldest methods of food preservation (Al-Mahasneh *et al.,* 2013; Chatta *et al*., 2018; Doymaz, 2011; Ismail, 2016; Silva *et al*., 2014). The objectives of drying include: safe storage over an extended period of time (Fargali *et al.,* 2008), ease of handling and reduction in cost of transportation (Cakmak and Yildiz, 2012; Doymaz, 2005, 2012; Torki-harchegani *et al.,* 2016;

Tripathy and Kumar, 2009).

Moreover, more than 80% of the food in the developing countries are produced by the small farmers. These food products are preserved through sun drying by the farmers, i.e., natural sun drying is being practiced (Murthy, 2009). It is known that there is no electricity to use refrigeration in many rural areas in the developing countries for the preservation of the food products. Therefore, the only food preservation method available for the rural people is sun drying technique as in the case for many areas of Africa, Asia, Caribbean, and Latin America (Blaise *et al*., 2009).

Therefore, open sun drying is simply defined as the drying process which consists of spreading the crop in a thin layer on the ground (Jain and Tiwari, 2003) and expose the crop directly to solar radiation, wind and other ambient conditions (C¸akmak and Cengiz, 2011; Fargali *et al*., 2008). Sun drying has been in practice since long time ago for preserving food and agricultural crops (Saleh and Badran, 2009). Although, there are several demerits with the open sun drying process which include: loss of produce due to animals and birds; spoilt products due to moisture, wind, rain and dust; deterioration in the harvested crops due to fungi, decomposition and insect attacks. Moreover, sun drying process is time consuming, labour intensive and requires large space for drying the produce (Dilip and Pathare, 2007; Mghazli *et al*., 2017; Oyerinde, 2016; Sahdev, 2014; Sahdev *et al.,* 2017; Sharma *et al.,* 2009).

However, with all these disadvantages, sun drying is still practiced by small farmers or rural people due to its low cost set up, its simplicity, requires little expertise and requires only direct sun light which is cheap, non-pollutant, environmental friendly, abundant and renewable (Hii *et al.,* 2008; Jayashree and Visvanathan, 2013; Karaaslan *et al.,* 2016).

Furthermore, Thin-layer drying can be described as the procedure of drying single layer of a particles or slices of a product (Blanco-cano *et al.,* 2016; Pandey *et al.,* 2015). Due to the thin-layer drying, the temperature distribution is considered uniform. The main mechanisms of drying include: surface diffusion on the pore surfaces which occurs during the constant rate period of drying; liquid or vapour diffusion due to moisture concentration differences which occurs during the falling rate period of drying and capillary action in granular and porous foods due to surface forces. Generally, hygroscopic materials dry in constant rate and subsequent falling rate periods and drying stops when equilibrium is reached (Pandey *et al*., 2015).

2.0 MATHEMATICAL MODELLING OF THIN LAYER DRYING

In mathematical modelling of drying, the thin-layer drying equations are important tools. The thin-layer equations have been used to estimate the drying time of many agricultural products, to simulate moisture movement and mass transfer during the drying of agricultural products, in the design of dryers and to generalized the drying curve (Aregbesola *et al.,* 2015; Jayashree and Visvanathan, 2013; Toĝrul and Pehlivan, 2004; Vijayan *et al.,* 2016). The thin-layer drying models that describe the drying phenomenon of agricultural products are usually categorized into three (3), namely: theoretical, semi-theoretical and empirical (Pandey *et al*., 2015; Sharada, 2013; Vijayan *et al*., 2016).

The theoretical models clearly describe the drying behaviours of agricultural products and can be used for all process conditions, but may include assumptions about moisture mechanisms which may cause considerable error (Pandey *et al*., 2015). One of the theoretical models used in drying is the Fick's second law equation which has been used widely for thin-layer drying process. Semi-theoretical models are generally derived from Fick's second law (Pandey *et al*., 2015). The empirical and semi-theoretical models consider external resistance to moisture transfer resistance between product and air but a theoretical model takes into account only internal resistance to moisture transfer. An empirical model gives a better fit to the experimental data without any understanding of the transport processes involved whereas the theoretical model gives a better understanding of the transport processes involved in the drying (Pandey *et al*., 2015). The most commonly used thin-layer drying models are given in the table below.

Table 1.1 models used in thin layer drying of agricultural products

Model number	Model name	Model	Reference
1	Newton	MR=exp(-kt)	(O'Callaghan *et al.,* 1971)
2	Page	MR = exp(−kt^n)	(Diamante and Munro, 1993)
3	Modified page	MR=exp[-(kt)^n]	(Overhults *et al.,* 1973; White *et al.,* 1978)
4	Henderson and Pabis	MR=a exp(-kt)	(Zhang and Litchfield, 1991)
5	Modified Henderson and Pabis	MR=a exp(-kt)+b exp(-gt)+c exp(-ht)	(Karathanos, 1999)
6	Two term	MR=a exp(k_0t)+b exp(-k_1t)	(Sharaf-Eldeen *et al.,* 1980)
7	Two term exponential	MR=a exp(-kt)+(1-a)exp(-kat)	(Sharaf-Eldeen et al., 1980)
8	Approximation of diffusion	MR=a exp(-kt)+(1-a)exp(-kbt)	(Yaldiz *et al*., 2001)
9	Verma et al.	MR=a exp(-kt)+(1-a)exp(-gt)	(Lalit *et al.,* 1985)

10	Logarithmic	MR=a exp(-kt)+c	(Yagcioglu & A., 1999)
11	Wang and Singh	$MR=1+at+bt^2$	(Wang and Singh, 1978)
12	Midilli & Kucuk	$MR= a.exp(-kt^n)+bt$	(Akpinar and Bicer, 2007)

2.1 MATHEMATICAL MODELLING OF THIN-LAYER SUN DRYING OF GRAINS

An experiment was conducted by Alves *et al*. (2019) on the drying of cowpea bean grain using solar energy. Drying was performed during the day with samples exposed to the sun; during the night, the samples were left on a laboratory bench. The cowpea bean grains at initial moisture contents of 65.42% d.b were dried to final moisture content of 6.73% d.b within 54.4 h (3270min). The mean values of temperature and relative humidity in the external environment were 26.92 ± 2.9 ºC and 67.87 ± 12.3%, respectively. The drying rates occurred at the falling rate period. The models Approximation of Diffusion, Page, Verma, Logarithmic and Two Terms were fitted to the drying kinetics and all of them fitted well to the experimental data, with coefficients of determination (R^2) higher than 0.98, mean square deviations (MSD) less than 0.01 and chi-square (χ^2) values lower than 0.0001. The effective diffusivity values were of the order of 10^{-11} m^2/s for the cowpea bean samples. In an experiment to study the drying behaviour of rough rice in Ivory Coast, Ahou *et al*. (2014) dried aromatic rough rice under open sun at three different seasons (i.e. small dry season S1, harmattan season S2 and great dry season S3). The temperature and relative humidity for the three seasons S1, S2 and S3 are 30.7^0C, 69.26%; 31.52^0C, 45.86% and 32.42^0C, 60.66% respectively. The rough rice was dried from an initial moisture content of 19% d.b to a final moisture content of 12% d.b. The drying data were fitted to ten different mathematical models. Among the models, the Two-term model was found to best explain thin layer open sun drying behaviour of the rice. The performance of these models was investigated by comparing the determination of coefficient (R^2), sum square error (SSE) and root mean square error (RMSE) between the observed and predicted moisture ratios. The results showed that the Two-term model was found to be the most suitable for describing drying curve of rough rice, in all study season, with R^2 SSE RMSE ranging respectively from 0.9931 to 0.9957, from 0.0041 to 0.0093 and from 0.0143 to 0.0215. In this study, it was noticed that value of diffusion coefficients for the examined samples changed between 8.34×10^{-12} and 4.52×10^{-11} m^2/s, and the activation energy was estimated to 68.255 Kj/mol.

Al-Mahasneh *et al*. (2013) investigated the drying kinetics of roasted green wheat under open sun using Fuzzy and conventional modelling. The assumption of open sun drying adequacy to prevent deterioration was tested in this study. In this experiment, green beans were roasted by exposing them to direct fire for about 15 minutes before sun drying. The roasted green beans at an average moisture content of 43.2% (w.b) were sun dried to a safest moisture content of below 10% (w.b) in 5.5 hours. During the drying experiment, the ambient temperature varied between 26 and 32◦C. The experiment occurred in falling rate period of drying. The drying data were fitted into eight common thin-layer drying models. Goodness of fit for each model was evaluated using coefficient of determination (R^2) and root mean square error (RMSE). Among the conventional thin layer drying models, the two-term exponential model was found to best fit the roasted green wheat sun drying data with R^2, and root mean square error values of 0.988 and 0.038, respectively. Fuzzy modelling, however, provided better modelling capabilities compared to conventional models. The effective diffusivity was also evaluated for roasted green wheat kernels and found to be 1.7×10^{-11} m^2/s. The results showed that open sun drying for 5.5 h was effective and adequate to reduce moisture content to a safe level and to prevent deterioration of this product.

2.2 MATHEMATICAL MODELLING OF THIN-LAYER SUN DRYING OF FRIUTS

Santos *et al*. (2018) evaluated the drying kinetics of passion fruit seeds exposed to three drying conditions: full sun, half shade and shade (laboratory). The passion fruit seeds were dried at the initial moisture content of 0.7 (d.b) to a final moisture content of 0.4 (d.b.) at the end of drying. The drying period was 7h for the three situations studied. The temperature was measured by means of a chemical thermometer (wet and dry bulb). Twelve (12) thin-layer models were fitted to the experimental data. For the validation of the drying equations, analysis of nonlinear regression of the mathematical models of drying to the experimental data was performed. The values of the coefficient of determination (R^2), mean relative error (P), estimated mean error (SE), and chi-square (X^2) were used as criteria to verify the fitting degree of the mathematical models studied. The Wang and Singh model was the model that best fit the experimental data, with R^2 values closer to 1, X^2 and SE closer to 0 and smaller P. The sun drying condition obtained a greater efficiency in the water removal of passion fruit seeds. An investigation on the sun drying behaviour of cornelian cherry fruits was conducted by Polatoğlu and Beşe (2017). The drying process of cornelian cherry took place in the falling rate period as shown by the drying rate curve. For explaining the thin layer drying kinetics of cornelian cherry, about twelve thin-layer mathematical models were used. The approximation of diffusion model was found to be the most appropriate model for the process. The Fick's diffusion model was used to calculate the effective moisture diffusion coefficients (D_{eff}) of cornelian cherry. The value of (D_{eff}) was obtained as 1.20×10^{-11} m^2/s. The vitamin C degradation of dried cornelian cherry was determined as about 51.1%.

Thin layer drying experiments were conducted by Olabinjo *et al*. (2017) to compute the drying characteristics of fermented cocoa beans in open sun and indirect natural convection solar dryer. The drying experiments were conducted at the same time for comparison. Three different thin layers drying of the fermented cocoa beans were examined under field conditions of Akure, Nigeria. The drying process took place only in the falling rate period. The drying curves obtained from the experimental data were fitted to thirteen (13) different thin layer mathematical models. All the models were compared according to three evaluation parameters. These include coefficient of determination (R^2), Root mean square error (RMSE) and Chi-square (X^2).The results showed that the Midilli and Kucuk model, best described the drying curve of fermented cocoa beans under open sun with R^2 = 0.9866, X^2=0.0024 and RMSE=0.0023. Olawoye *et al*. (2017) performed an experiment on Modelling of thin-layer drying characteristic of unripe Cardaba banana (Musa ABB) slices. The drying kinetics of Carbada banana were investigated using sun and hot air-drying at the temperature. The drying rate of the convective hot air oven was kept constant at 1.2 m^2/s. The Cardaba samples had a thickness of 5 mm and the initial moisture content of about 79.80% w.b (395.05% d.b) were dried to the final moisture content of 13.61% d.b for drying using sun. The drying curves of the Cardaba banana slices contained no constant rate of drying but took place in the falling rate period. Twelve thin-layer drying models were used to fit the drying data and compared according to their coefficients of determination (R^2), root mean square error (RMSE) and reduced χ^2 to estimate drying curves. Among the thin-layer models, Wang and Singh model with R^2 of 0.9953, RMSE of 0.0223 and χ^2 of 4.94×10^{-4} was found to best explain the drying behaviour of the Cardaba banana slices. Effective moisture diffusivity of Cardaba banana slices increased from 1.46×10^{-8} to 4.25×10^{-8} m^2/s, resulting to activation energy of 38.46 kJ/mol.

The drying kinetics of pineapple was studied by Olanipekun *et al*. (2015), and the model that best describes it was selected. Pineapple slices were dried in a hot-air oven, microwave and under direct sunlight. Drying took place entirely in the falling rate-drying period. Seven mathematical models were fitted into the experimental data. The goodness of fit was determined using the coefficient of determination (R^2), reduced chi-square (χ^2), root mean square error (RMSE) and sum square error (SSE). The Page model best explained the sun-drying behaviour of the pineapple. The effective moisture diffusivity increased from 6.89×10^{-10} to 5.1×10^{-8} m^2/s. Doymaz (2012) investigated the sun drying behaviour of seedless and seeded grapes. The drying study showed that the times taken for drying of seedless and seeded grapes of berry size of 1.72 cm and 2.20 cm thicknesses from the initial moisture contents of 78.2% and 79.5% (w.b.) to final moisture content of around 22% (w.b.) were 176 and 228 h in open sun drying, respectively. The temperature of ambient air ranged from 32 to 46 °C. The drying data were fitted to 12 thin-layer drying models. The performance of these models were compared using the determination of coefficient (R^2), mean relative percent error (P), reduced chi-square (χ^2) and root mean square error (RMSE) between the observed and predicted moisture ratios. The results showed that Midilli et al. model was found to satisfactorily describe the sun drying curves of seedless and seeded grapes. The effective moisture diffusivity values were estimated from Fick's diffusion model by 1.02×10^{-11} and 1.66×10^{-11} m^2/s for seeded and seedless grapes.

2.3 MATHEMATICAL MODELLING OF THIN-LAYER SUN DRYING OF VEGETABLES

The thin layer drying behaviour of groundnut was investigated under open sun drying (OSD) and indoor forced convection drying (IFCD) modes by Sahdev *et al*. (2017). The groundnut samples were dried from initial moisture content of 38% (w.b.) to the safe storage moisture content of 8%-10% (w.b.). Four mathematical models were compared for describing the groundnut drying process. The performance of thin layer drying models was investigated by comparing the statistical parameters such as coefficient of correlation (R), reduced chi-square (χ^2), root mean square error (RMSE), and mean bias error (MBE) between experimental and predicted moisture ratios. Henderson and Pabis model was observed to give the highest value of R and lowest values of χ^2, RMSE and MBE for the groundnut drying under both OSD and IFCD modes. The values of statistical parameters under Lewis model were also found to be very close to Henderson and Pabis model. Therefore, Henderson and Pabis and Lewis models were found to be the best for describing the drying behaviour of groundnut under both given drying conditions. Ismail (2016) investigated the effects of open-air sun drying and pre-treatment on drying characteristic of cherry tomato (Lycopersicum esculentum). The sun drying times were determined as 26h for natural cherry tomato and 22 h for pre-treated cherry tomato samples. The pre-treated cherry tomato samples were dried in slightly shorter time than the natural cherry tomato samples. There were no any constant rate drying period occurred, so the drying process took place in the falling rate period. About seven theoretical drying models were applied to experimental data and all the models were compared according to statistical parameters; i.e. coefficient of determination (R^2), chi-square (χ^2) and root mean square error (RMSE). Among the seven models used, it was observed that Verma et al. model is the best mathematical model represented the open-air sun drying behaviour of cherry tomatoes. Based on Fick's second law, the values of effective moisture diffusivity for treated and natural cherry tomatoes samples are found to be 4.76×10^{-10} and 4.42×10^{-10} m^2/s respectively.

Oyerinde (2016) conducted drying experiment using direct sun drying and indirect passive solar dryer to simulate the drying processes of tomato slices. Tomato slices of 3mm thickness were placed on perforated stainless steel trays in a thin layer and dried to equilibrium moisture content. All samples were dried from an initial moisture content of 95.4 %w.b to 10.2 %w.b for sun dried samples and 8.5 %w.b for solar dried samples. Ambient temperature (dry bulb) varied from 22.0 ^{0}C – 32.5 ^{0}C and relative humidity from 70.8 % – 97.0 %, while solar radiation varied from 231.14 W/m^2 – 912.41W/m^2 during the drying period. Drying time was 15 hours over a period of 3 days for sun dried samples. To explain the drying characteristics of tomatoes slices, ten semi-theoretical and empirical models found in literature were applied and fitted to the experimental data. From the statistical analysis, it was concluded that the Page model best predicted the drying data for both sun and solar dried samples. Effective moisture diffusivity was $5.07\times10^{-7}m^2/s$ for sun dried samples, while activation energy ranged from 32.38 to 33.53 kJ/mol for samples dried under open sun drying. Fadhel *et al*. (2014) studied and analysed the drying of red pepper known as "Baklouti" by three different solar processes. These three drying processes include: natural convection solar drier, greenhouse and open sun. During the drying experiments, the temperature of ambient air ranged from 18.21 to 33.57°C. On the other hand, the solar radiation ranged from 812 to 902 W/m^2 and the relative humidity of ambient air ranged from 53.5 to 91.9%. Drying time (including nights) is about 118 hours in open sun. Six thin-layer drying models (Newton, Henderson and Pabis, Modified Henderson and Pabis, Wang and Singh, Logarithmic and Two- term) were fitted to the experimental data to select a suitable drying equation. The Logarithmic was found to best describe the drying behaviour of pepper for open sun, greenhouse and solar drier drying. Fudholi *et al*. (2013) evaluated the drying of the Malaysian red chili (Capsicumannuum L.) under open sun and solar drying processes. Red chilies were dried down from approximately 80% (wb) to 10% (wb) moisture content within 33h using solar dryer and 65h using open sun drying. The drying process was conducted during the day at the average solar radiation of 420W/m^2 and air flow rate of 0.07 kg/s. Solar drying yielded a 49% saving in drying time compared with open sun drying. A nonlinear regression procedure was used to fit three drying models. These models were compared with experimental data on red chilies dried by open sun drying and those dried by solar drying. The fit quality of the models was evaluated using their coefficient of determination (R^2), mean bias error (MBE), and root-mean-square error (RMSE) values. The Page model resulted in the highest (R^2) and the lowest (MBE) and (RMSE) which make it the best fit.

The effect of sample thickness, method of drying and drying air temperature on the drying characteristics and kinetics of okra slices were investigated by Afolabi and Agarry (2014). The samples (10mm and 20mm thick) were dried under open sun and solar dryer. The okra slices dried perfectly within 216 – 240 h, under open sun. The samples dried in the falling rate period with no constant rate period. Four thin-layer semi- empirical mathematical drying models (Newton, Page, Henderson and Pabis, and Logarithmic models) were fitted to the experimental drying curves. The models were compared using the coefficient of determination (R^2) and the root mean square error (RMSE). The logarithmic model has shown a better fit to the experimental data obtained as relatively compared to other tested models. The transport of water during drying was described by application of Fick's diffusion model and the effective moisture diffusivity was estimated. The value ranges from 0.253 to 0.901 × 10-10 m^2/s for open sun drying. Jayashree and Visvanathan (2013) conducted an experiment titled "Mathematical modelling for thin layer sun drying of ginger (Zingiber officinale Rosc.)". In the experiment, ginger were dried during the month of April, 2009 at Agricultural Engineering College and Research Institute, Coimbatore (Tamil Nadu). Ginger rhizomes were mechanically washed, partially peeled, spread in single layer on cemented yard and dried from initial moisture of 594.01% (d.b.) (dry basis) to a final moisture content value of 9.82% (d.b.). During the days of the experiment, the temperature of the drying air varied from 30.3°C to 38.1°C while the average relative humidity varied from 62.47% to 35.09%, which corresponded to the time when the average solar intensity obtained was maximum (889.38 W m^2) and the average wind speed varied from 0.5 m/s to 1.3 m/s. Drying of ginger was completed in eight days. Drying characteristics curves, showed no constant rate period and all the drying process occurred in the falling rate period. Thin layer modelling of drying data showed that diffusion approximation model best described the drying process. The effective moisture diffusivity for drying of ginger was calculated as 1.91×10^{-7} m^2/s. Sun dried ginger rhizomes were evaluated for its quality and it was found that the essential oil, oleoresin, moisture and crude fibre contents were 2.0%, 4.6%, 9.82% and 2.5%, respectively.

3.0 Mathematical modelling approach

The mathematical modelling of drying agricultural product is done based on the drying characteristics obtained from the experimental investigation. To get the drying characteristics of the product, thin layer drying requires some important parameters which include the following(Pandey *et al*., 2015);

3.0.1 Important parameters

3.0.1.1 Moisture content

The quantity of moisture present in a material can be expressed either on the wet basis or dry basis and expressed either as decimal or percentage. The moisture content on the wet basis is the weight of moisture present in a product per unit weight of the undried material, represented as,

$$M_{wb} = \frac{W_0 - W_d}{W_0} \quad (1)$$

Percentage wet basis (wb) is expressed as,

Percentage M_{wb} = M_{wb} x 100 2

While the moisture content on the dry basis is the weight of moisture present in the product per unit weight of dry matter in the product and represented as,

$$M_{db} = \frac{W_0 - W_d}{W_d} \quad (3)$$

Percentage dry basis (db) is expressed as,

Percentage M_{db} = M_{db} x 100 4

The moisture contents on the wet and dry basis are inter-related according to the following equations,

$$M_d = \left(\frac{M_w}{100 - M_w}\right) \times 100 \quad (5)$$

$$M_w = \left(\frac{M_d}{100 + M_d}\right) \times 100 \quad (6)$$

The moisture content on the wet basis is used normally for commercial purposes, while the moisture content on the dry basis has tended to be employed for engineering research designation, because the weight change associated with each percentage point of moisture reduction on the dry basis is constant as against the wet basis where the amount of water involved in a moisture content reduction of one percent changes as drying progresses, because the weight of water and total crop weight change.

3.0.1.2 Equilibrium moisture content (Me)

A crop has a characteristic water vapour pressure at a particular temperature and moisture content. The equilibrium moisture content is the moisture content at which the product is neither gaining nor losing moisture. It is a dynamic equilibrium which changes with relative humidity and temperature.

3.0.1.3 Moisture ratio (MR)

Moisture ratio is one of the important criteria to determine the drying characteristics of agricultural product. MR can be determined according to external conditions. If the relative humidity of the drying air is constant during the drying process, then the moisture equilibrium is constant too. In this respect, MR is determined as in Eq. (7).

$$MR = \frac{M_t - M_e}{M_o - M_e} \qquad 7$$

If the relative humidity of the drying air continuously fluctuates, then the moisture equilibrium continuously varies so MR is determined as in Eq.(8) given by (Diamante and Munro, 1993).

$$MR = \frac{M_t}{M_o} \qquad 8$$

3.0.1.4 Drying rates

Drying rate may be defined as the amount of moisture lost per unit of drying surface area per unit of drying time. This can be expressed as;

Drying rate (R) =

$$\frac{M_{t+dt} - M_t}{dt} \qquad 9$$

3.0.2. Periods of Drying

There are two main drying rate regimes for agricultural products, namely the constant drying rate period and the falling drying rate period.

3.3.2.1 Constant drying rate period

During the constant drying rate period, drying takes place from the surface of the product and is simply the evaporation of moisture from the free-water surface. The rate of moisture removal during this period is mainly dependent on the surrounding conditions and only affected slightly by the nature of the product. At the end of the constant drying period is the critical moisture content.

3.3.2.2 Falling drying rate period

Below the critical moisture content is the falling drying rate period. This drying rate regime is dependent essentially on the rate of diffusion of moisture from within the product to the surface and also on moisture removal from the surface.

Figure 1 shows a plot of moisture content (W) versus time (θ) generally obtained by experimentally drying a solid. This curve represents a typical case when a wet solid loses moisture initially by evaporation from a saturated surface on a solid, followed by a period of evaporation from a saturated surface of gradually decreasing area and finally when the latter evaporated in the interior of the solid. Figure 1a indicates that the drying rate is subject to variation with time or moisture content, further better illustrated by graphically or numerically differentiating the curve and plotting *dW*/*d*θ versus *W,* as shown in Figure 1b*,* or as *dW*/*d*θ versus θ, as shown in Figure 1c*.* These rate curves illustrate that the drying process is not a smooth, continuous one in which a single mechanism controls throughout. Figure 1*c* has the advantage of showing how long each drying period lasts.

The section *AB* on each curve represents a warming-up period of the solids. Section *BC* on each curve represents the constant-rate period. Point *C,* where the constant rate ends and the drying rate begins falling, is termed the critical-moisture content. The curved portion *CD* on Figure 1a is termed the falling-rate period and, as shown in Figure 1*b* and *c,* is typified by a continuously changing rate throughout the remainder of the drying cycle. Point *E* (Figure 1b) represents the point at which all the exposed surface becomes completely unsaturated and marks the start of that portion of the drying cycle during which the rate of internal moisture movement controls the drying rate. Portion *CE* in Figure 1.1b is usually defined as the first falling-rate drying period; and portion *DE,* as the second falling-rate period (Perry, 2007).

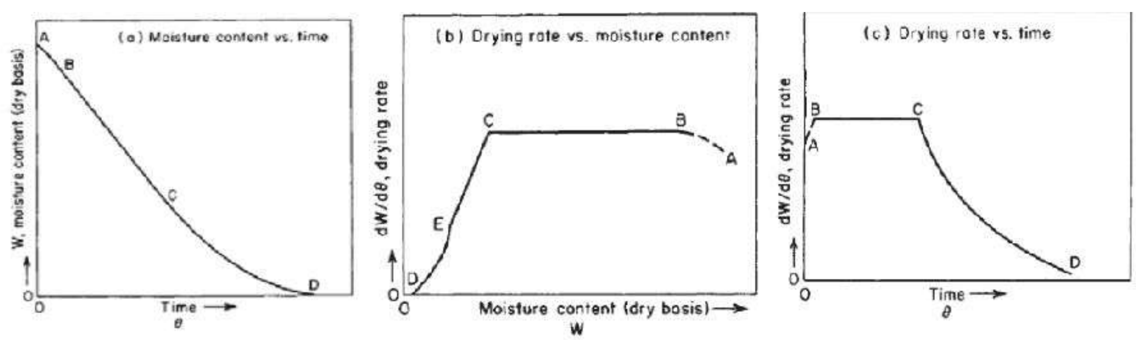

Figure 1 drying period curves (Perry, 2007)

3.1 Determination of appropriate model

Mathematical modelling of the drying of food products often requires the statistical methods of regression and correlation analysis. Linear and nonlinear regression analyses are important tools to find the relationship between different variables, especially, for which no established empirical relationship exists. Thin layer drying equations require MR variation versus time (t). Therefore, MR data plotted with time (t) and regression analysis is performed with the selected models to determine the constant values that supply the best appropriateness of models. The validation of models can be checked with different statistical methods. The most widely used method is determining correlation coefficient (r), coefficient of determination (r^2), reduced chi-square (χ^2) test and root mean square error (RMSE) analysis (Pandey et al., 2015).

Correlation coefficient (r)-

$$r = \frac{\sum_{i=1}^{N} MR_{pre,i}\, MR_{exp,i} - \sum_{i=1}^{N} MR_{pre,i} \sum_{i=1}^{N} MR_{exp,i}}{\sqrt{\left(N\sum_{i=1}^{N} (MR_{pre,i})^2 - \left(\sum_{i=1}^{N} MR_{pre,i}\right)^2\right)\left(N\sum_{i=1}^{N} MR_{exp,i} - \left(\sum_{i=1}^{N} MR_{exp,i}\right)^2\right)}} \qquad 10$$

Coefficient of determination (r^2)- (Akpinar, 2008)

$$R^2 = \frac{\sum_{i=1}^{n}\left(MR_i - MR_{pre,i}\right) \cdot \sum_{i=1}^{n}\left(MR_i - MR_{exp,i}\right)}{\sqrt{\left[\sum_{i=1}^{n}\left(MR_i - MR_{pre,i}\right)^2\right] \cdot \left[\sum_{i=1}^{n}\left(MR_i - MR_{exp,i}\right)^2\right]}} \quad (11)$$

Reduced chi-square (χ^2)-

$$X^2 = \frac{\sum_{i=1}^{n}(MR_{exp,i} - MR_{pre,i})^2}{N-n} \quad (12)$$

Root mean square error (RMSE)-

$$RMSE = [\sum_{i=1}^{N} \frac{1}{N}(MR_{exp,i} - MR_{pre,i})^2]^{\frac{1}{2}} \quad (13)$$

The highest correlation coefficient (r) and coefficient of determination (r^2), and the lowest chi square (X^2) and root mean square error (RMSE) values are required to select the best suitable model to explain the thin layer drying process. Once the drying curves obtained from the experimental data, then it will be fitted to the different semi-theoretical thin layer drying models (shown in table 1.1). The model which satisfies these requirements will be selected to represent the thin layer behaviour of the product.

4.0 Conclusions

A comprehensive review of the fundamental principles and theories required for the mathematical modelling of thin layer drying has been presented. For different agricultural product, different thin layer model was selected as per the statistical approach.

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Nomenclature

a, b, c, g, h, n	Empirical constants in the drying models
k, k_0, k_1	Empirical constants in the drying models
n	Number of constants
N	Number of observations
MR	Moisture ratio
MRexp	Experimental moisture ratio
MRpre	Predicted moisture ratio
M	Moisture content, (% dry basis)
Me	Equilibrium moisture content, (% dry basis)
M_0	Initial moisture content, (% dry basis)
Mt	Moisture content at t, (% dry basis)
M_{t+dt}	Moisture content at t+dt, (% dry basis)
t	Time, (min or hour)
T	Temperature, (^{0}C)
T_{abs}	Absolute temperature, (^{0}K)

Abbreviations:

wb:	Wet Basis
db:	Dry basis
RMSE:	Root mean square error

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Innovative Approaches in Teaching Chemistry in Digital Era at Secondary School Level in Nigeria: Issues and Prospects

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Abstract: *Teaching Chemistry using innovative approaches cum digital devices is one of the most viable strategies, notwithstanding the attendance challenges. This paper examined pedagogical techniques and interactive teaching in the digital era. The paper delved into teaching chemistry where Students are actively involved in laboratory activities, such as experimentations, problem solving and other related Methods. The effectiveness of utilizing equipment, Process Oriented Guided Inquiry Learning Approach (POGILA) in teaching Chemistry were highlighted. Similarly, innovative techniques and key process skills such as ability to think analytically and work effectively as part of collaborative learning were examined. The paper viewed innovative approaches in teaching as improved problem solving skills which allows students to work at their pace and systematically master the concepts. The paper also highlighted the concept of digital era, techniques and devices used in teaching Chemistry. Issues and prospects were equally highlighted as they affect teaching and learning chemistry. The paper recommended among others provision of qualified chemistry teachers and adequate facilities as well as internet resources for teaching of chemistry.*

Keywords*: Approaches, Chemistry, Digital, Innovative, Teaching*

Introduction

The teaching of Chemistry involves innovative approaches which are pedagogically oriented Student-centered. Interactive teaching in Science is based on inclusive approaches' that are research based which provide students with meaningful learning experiences in Chemistry. Chemistry as a science subject offers unique opportunity for leaning about how science works and interact with other learning experiences in life and society (Eilks and Hofstein, 2013). Learning Chemistry using innovative approaches helps in developing general skills, such as problems solving, understanding how Science contributes to society 's sustainable development, thinking in models being sensitive to and aware of dangers and hazards for environmental protection. In this way, chemistry has the potential to contribute to developing general education skills. Some of the skills can be developed with other Science subjects but of these skills are only unique to Chemistry.

It is worth nothing that teaching chemistry as a subject offers Student the opportunity to acquire knowledge about the world around them, enable them to contribute in societal debate about science and technology related issues. Hence, the need for innovative approaches in teaching Chemistry that brings about desired outcome. Eilks and Hofstein (2013) observed that despite the apparent and conspicuous centrality of Chemistry among the science disciplines, its demanding goals have not been satisfactorily achieved. However, with the emergence and integration of E-learning approaches in science, achievement of the objectives is enhanced. It is against this background that this paper will focus on the following objectives:

- Concept of Chemistry.
- Concept of Digital era.
- Innovative approaches in teaching chemistry.
- Issues in teaching chemistry.
- Prospects in teaching chemistry.
- Suggestions.

Concept of Chemistry

Chemistry is a branch of physical science which deals with structure, properties and changes of matter (Wikipedia, 2011). Chemistry includes topics such as the properties of individual atoms, how atoms form chemical bonds to create chemical compounds, the interactions of substance through intermolecular forces that give matter its general properties, and interactions between substances through chemical reaction to form different substance. Conceptually, the definition of chemistry kept changing over the years due to researches, innovations and new approaches in teaching and learning of the subject. Eilks and Hofstein (2013) noted that the history of chemistry education is a culture of dynamic innovation characterized by new approaches cum knowledge and skills. This of course included new methods, strategies and techniques of teaching. Colgoni and Eyles (2010) Conceptualized science subjects in general and chemistry in particular as broad based interdisciplinary programmes that need a range of integrated approaches. In this context chemistry is regarded as an essential basis for many facts of our everyday lives, and has many unforeseen potential benefits for our future. Proper understanding of chemistry enables us to explain the world around us. This explains the basic necessity to teach and learn chemistry which knowledge and skills remain dynamic in the face of the changing nature of the society. Accordingly, the approaches, methods and techniques of teaching chemistry keep on changing. This calls for the need to explain innovation approaches in teaching chemistry.

Concept of Digital Era

Digital era can be conceptualized in terms of information age within a period in human history characterized by the shift from traditional to industrial revolution. It is worth nothing that the definition of digital era continuous to change over time. In this context the concept of digital era refers to e-learning. Digital learning, therefore, refers to any type of learning that is facilitated by teaching or by instructional practice that make use of technology. Digital learning occurs in all learning areas or domains. According to Victoria (2017) it encompasses the application of a wide spectrum of practices which include:

- Blended and digital learning

- Game based learning
- Assessing digital context
- Collaborating locally and globally
- Assessing and reporting on time

Active participation in online communities
Using technology to connect Collaborate, curate and create

Innovative Approaches in Teaching Chemistry in Digital Era

The concept of innovative approaches has to do with new methods techniques, strategies and skills in teaching. Nicolaides (2012) conceptualized innovative approaches in terms of multimedia technologies that are available for developers. This view represents multimedia devices that are interactive and resourceful in teaching. Similarly, Ogbondahi (2008) conceptualized multimedia devices in terms of three categories: visual, audio-visual and aural devices. These devices provide avenues digitalization in teaching and learning process.

The above views represent mainly the concept of teaching materials which are basically features and constituent of techniques and skills of teaching and learning process. Degereji (2006) referred to approach as any viable practical method of teaching that brings about adesired outcome. This view represents innovative approaches as any pedagogical and interactive teaching techniques that activity involved learners. Similarly, Kabutu and Degereji (2006) noted discovery and problem solving as viable approaches in teaching broad base science which allows students to actively participate in their own observations.

Eilks *et al*. (2013) referred to innovative approaches in teaching chemistry as meaningful contexts to motivate the learning of chemistry. This usually stem from the structure of the discipline or history of chemistry, via everyday life contexts, industrial applications, or environmental issues and socio scientific issues. They further maintained that general orientations offer textual approaches to start the lesson, but the orientations can be used as guiding principles for structuring the whole chemistry curriculum. These orientations of approaches are;

- History of science (chemistry) orientation;
- Structure of the discipline orientation;
- Everyday orientation;
- Environmental orientation;
- Technology and industry orientation;
- Socio-scientific issues orientation.

The above orientation textual approaches can be used within the context based chemistry curriculum. Innovative approaches are eminent due to growing awareness about the problem in traditional chemistry teaching. Eilks *et al*. (2013) noted that orientation on contexts means topics are chosen as the basis from which to start chemistry teaching and learning. In innovative approaches, contexts are thought to engage the students and provoke questions in this regards, connection to basic concepts ensures that the chemistry knowledge students gained within an individual context is detached from the specific context. The decontextualization and networking leads to cumulative learning of the basic concepts such as context on “food” provokes questions which answer lead to certain chemical knowledge. This knowledge is elaborated upon in a variety of ways until the questions are answered. For example, the elaboration of a context on burning will use some of this knowledge and produce some more. Using innovative approaches methods and techniques more knowledge is built up and whenever elements of a basic concept emerge, they are reflected and used for systematic organizing of the acquired knowledge.

Schwartz *et al*. (2013) noted in using innovative approaches in teaching chemistry. A teacher may ask the following questions: What do I want my students to understand? Or; What are they supposed to be able to do as a result of learning?In an attempt to answer these questions in the event of teaching and learning process, chemistry students should be engaged in active learning using variety of methods and techniques.Colgoni and Eyles (2010) and Eilks and Hofstein (2013) maintained that Innovative approaches such as process oriented guided inquiry learning, laboratory technique, electronic learning, mattering chemistry and smart work among others are innovative research- based online environment designed for both effective teaching and learning, similarly, concept simulations tutorials, visual experiences and problems based home, improve student learning and performance in chemistry. Students should be exposed to laboratory activities so that they learn about chemistry methods and experimentation. These classroom and laboratory techniques help in teaching content and key process skills such as ability to think analytically and work effectively as part of collaborative and interactive learning.

The innovative approaches in teaching science also encapsulate digital techniques chemistry. For example, a teacher can use video project to teach global warming for students to benefit maximally from the knowledge and skills. Lessley (2014) and Eilks *et al*. (2009) noted the importance of the following digital techniques in teaching chemistry: Video games, Team games, Role playing, question games, Puzzles, Discussion, Table top games, lets imagine games and Quizzes, these techniques can be digitalized and enhanced teaching chemistry. Different types of technology can be used in the digital era to teach in the classroom. In this regard, Karehka (2012) noted that use of computer assisted learning in the classroom, smart interactive, white board, online media and online study tools are very effective in teaching various subjects.

Issues in Teaching Chemistry

Chemistry as an academic discipline or subject involves the teachings of theory and practical concepts. In this regard, it requires both human and material resources such as qualified teachers and instructional facilities.

Issues related to teaching of chemistry according to Edomwonyi-Out and Abraham (2011) includes attitudes of both teachers and students, non-professionalism, time constraints, work shop, class size, conditions of service, laboratory adequacy and examination malpractice. These are among critical current issues of teaching chemistry in the digital era. Ouma (2011) examines these issues in terms of factors influencing students’ academic performance in chemistry and emphasized their centrality Vis-a-Vis digitalization of teaching chemistry.

In the light of the above issues in teaching chemistry can feature in two dimension: availability of teaching resources and technical know-how of the teacher in using the resources. This of course does not negate the creativity and resourcefulness of the teacher in making sure that acquiring knowledge based on the digital technology goes hand in hand with the knowledge of the subject matter.

Funding is one of the critical issues affecting the teaching of chemistry, particularly the practical aspects. It is obvious that provision of quality resources determines quality education. Training and retraining of teachers and adequate provision of teaching and learning resources depends on the availability of funds. In Nigeria there are broadly two sources of funding: Government and non-governmental (Famade *et al*., 2015). In this context, huge amount of money is required for provision and maintenance of digital learning resources. These may be more obvious in subject like chemistry which teaching is not only based on use of available materials but research based instructional technology such as video programmes, video games, and internet resources among others.

The issue of Power supply is also central to the teaching and learning of different concept in chemistry. Use of e- learning resources and other digital devices squarely depend on power supply.

Prospects in Teaching and learning Chemistry

The prospects in teaching and learning chemistry are numerous. Carson (2012) maintained that teaching chemistry using instructional technology helps students to study in their own time and wide range of fields which boost level of self-motivation. E- learning strategies are effective because students can finish learning task in a short time frame, thereby creating time for more things to be learned. Using e- learning devices also enables students assessed all resources of a traditional course which of course helps students to learn where ever they found themselves. Armstrong (2013) noted that instructional technology through the use of digital devices allow students the freedoms to choose the time for study. Equally, it is important to note that digital learning resources enforce self-discipline among students**.**

Suggestions

This paper puts in place the following suggestions towards improving teaching and learning of chemistry in the digital era:

- Government should ensure that adequate facilities and internet resources are provided to improve teaching and learning chemistry.
- There is need for both government and private sectors to engage adequate qualified teachers who are willing to do the job.
- The present status of power supply is somehow encouraging; however, there is need to improve on the current situation by the government.
- Provision of e-learning resource is capital intensive; thus the government needs to prioritize provision of funds to enable schools used the digital technology effectively.
- There is need for government to ensure that chemistry teachers are professionally trained and re-trained to improve their knowledge and skills in teaching.

- Government should encourage research and implementation of research findings to improve chemistry teaching and learning process.
- Chemistry teachers be encouraging to employ the use of research-based teaching

Conclusion

This paper briefly defined the concept chemistry and examined innovative approaches as well as digital devices in teaching chemistry. The paper described pedagogical techniques within the context of innovative approaches and applicable digital technology in teaching chemistry as interactive teaching laboratory activities, scientific arguments and effective utilization of equipment vis-à-vis innovative skills and key process. Similarly, the paper highlighted some key digital devices. Thus the paper recommended among others the need for government to provide adequate teaching and learning resources in chemistry.

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